Math 8: Homework #7 Solution

3.1 - 4 a

a) $A = \emptyset$, $C = \{1\}$, $D = \emptyset$, $B = \{2\}$

3.1 - 9 a,c,g

a) $R \circ S = \{(3, 5), (5, 2)\}$

c) $T \circ S = \{(2, 1), (3, 1), (3, 4)\}$

g) $R \circ (S \circ T) = R \circ \{(3, 5)\} = \{(3, 2)\}$

3.1 - 20 b, c, g

b) $(a, c) \notin BXD \Rightarrow a \notin B \vee c \notin D$

c) AXB is cartesian product, not multiplication

g) If $(x, z) \in R$ and $(y, z) \in R$, then it is not necessarily true that $x = y$

3.2 - 1 d, i, j

d) This is clearly not reflexive $a < a$ is not true. Also not symmetric since if $a < b$, then $b < a$ is false. This is transitive however.

i) This is not reflexive since l cannot be perpendicular to itself. It is symmetric. It is not transitive.

j) This is not reflexive (try (0,-1)). This is symmetric. This is not transitive (try (0,0), (1,2), (3,2)).

3.2 - 4 e, j

e) Since $x^2 + y^2 = x^2 + y^2$, this relation is reflexive. Since $x^2 + y2 = a^2 + b^2 \Rightarrow a^2 + b^2 = x^2 + y^2$, then this relation is symmetric. And since if $x^2 + y^2 = a^2 + b^2$ and $a^2 + b^2 = c^2 + d^2$, then $x^2 + y^2 = c^2 + d^2$, this relation is transitive. So therefore this relation is an equivalence relation. The equivalence classes of $(1, 2)$ and $(4, 0)$ are circles of respective radii $\sqrt(5)$ and 4 centered at the $(0, 0)$.

j) This relation is reflexive since $f' = f'$. This relation is symmetric because if fRg then $f' = g'$, so $g' = f'$, therefore gRf. This relation is also transitive. If fRg and gRh, then $f' = g'$ and $g' = h'$, so $f' = h'$, so fRh. Therefore this relation is an equivalence relation. 3 elements from x^2/R are: $x^2, x^2+1, x^2 = 2$. 3 elements from $(4x^3+10x)/R$ are: $4x^3 + 10x, 4x^3+10x+2, 4x^3+10x+37$. x^3/R are functions of the form x^3+c where c is a constant. $7/R$ are any constant function.

3.2 - 7 c

c) $\overline{x} \bigcap \overline{y} = \emptyset \Rightarrow \exists p$ such that $x \equiv_m p$ and $y \equiv_m p$. Since $\equiv_m$ is an equivalence relation, then $p \equiv_m y$ by symmetry. And by transitivity $x \equiv_m y$. So $\overline{x} = \overline{y}$

3.2 - 8

a) R is reflexive since for all $x \in \mathbb{N}$, $x+x = 2x$ is divisible by 2. Suppose that $x+y$ is divisible by 2, then clearly $y+x$ is divisible by 2, so R is symmetric. If $x+y$ is divisible by 2 and $y+z$ is divisible by 2, then $x+y = 2q$ for some q and $y+z = 2r$ for some r. So $x+z = 2q-y+2r-y = 2(q+r-y)$. So $x+z$ is divisible by 2. So R is transitive. Therefore since R is reflexive, symmetric, and transitive, it is an equivalence relation.

b) S is not reflexive, $1+1$ is not divisible by 3. S is symmetric however since if $x+y$ is divisible by 3, so is $y+x$. However, S is not transitive, since $1+2$ is divisible by 3 and $2+1$ is divisible by 3, but $1+1$ is not divisible by 3. So since S is not reflexive, symmetric, and transitive, it is not an equivalence relation.

3.2 13

Since R is symmetric and transitive on A. Take any $a \in A$, then there is a $b \in A$ such that aRb. Now since R is symmetric, then bRa. And since R is transitive, then by combining the above, aRa. So R is reflexive on A.