Summary of Power Series, Maclaurin and Taylor Series, Fourier Series, and PDE's

Power Series:

Definition 1. *A power series is a series of the form* $\sum_{k=0}^{\infty} c_k x^k$*, or more generally:* $\sum_{k=0}^{\infty} c_k (x - x_0)^k$.

We would like to know which x's we can plug in to get a convergent series. To determine this, we consider the ratio test for power series:

To determine the interval of convergence of a power series:

1. Set $\rho = \frac{|u_{k+1}|}{|u_k|}$ and take the limit as $k \to \infty$.
2. Choose the x that make this limit less than 1.
3. This will give you a certain open interval (or possibly just $x = 0$ or $x = x_0$) where the series converges, but you aren't done yet.
4. You must now check the endpoints by plugging them into the series and seeing if the resulting series is convergent or not.

That's all there is to it.

Maclaurin and Taylor Series:

Definition 2. *If f is an infinitely differential function at $x = x_0$, then* $\sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$ *is called the Taylor series for f about $x = x_0$. If $x_0 = 0$, this is called the Maclaurin series.*

To find a Maclaurin or Taylor series:

1. Calculate the necessary derivatives and plug in x_0 and look for a pattern so that you can write out the series.
2. Although you can always obtain the Maclaurin or Taylor series by doing step 1, sometimes it is easier to derive a Maclaurin or Taylor series for a function from one of the known Taylor or Maclaurin series.

Some Important Maclaurin series:

1. $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$ when $-1 < x < 1$.

2. $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ for all x.

3. $\sin x = \sum_{k=0}(-1)^k \frac{x^{2k+1}}{(2k+1)!}$ for all x.

4. $\cos x = \sum_{k=0}(-1)^k \frac{x^{2k}}{(2k)!}$ for all x.

<u>Fourier Series:</u>

Definition 3. *For a function f defined on $[-L, L]$, the Fourier series for f is:* $a_0 + \sum_{k=1}^{\infty}(a_n \cos(\frac{n\pi x}{L}) + b_n \sin(\frac{n\pi x}{L}))$ *where* $a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$, $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos(\frac{n\pi x}{L})dx$, *and* $b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin(\frac{n\pi x}{L})dx$.

When you are asked to come up with a Fourier series for a function, all you need to do is evaluate the integrals in the definitions of a_0, a_n, and b_n and plug these into the Fourier series.

Some things to consider:

1. If f is even ($f(x) = f(-x)$), then $b_n = 0$ for all n.

2. If f is odd ($f(-x) = -f(x)$), then $a_0 = a_n = 0$ for all n.

<u>PDE's:</u>

To solve a PDE with boundary and initial conditions:

1. Use separation of variables to obtain 2 ODE's. (Set $u(x,t) = F(x)G(t)$, and plug this into the PDE. Then you try to get all F's on one side and all G's on another).

2. Apply any zero boundary or initial conditions to obtain solutions to the ODE's above.

3. Use Fourier analysis to make sure your solution satisfies any non-zero boundary or initial conditions.