Base 18, Quaternions, Markov Chains, and Absurdity

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2/16/11

Outline

1. The Backdrop
2. EAT ME
3. Hookahs and Mushrooms
4. The Mad t Party
5. Jabberwocky

The Backdrop

Figure: Queen Victoria

- Victorian Era: 1832-1901
- Prosperity in Industry
- Frumpy in Royalty
- Chloroform in Hospitals
- Children in Chimneys
- ¿Symbolic Algebra?

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"No word nor sign of arithmetic or algebra has one atom of meaning throughout this chapter, the object of which is *symbols, and their laws of combination*."

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ALICE: "I don't believe there's an atom of meaning in it."

The Real Alice and Her Sisters

Figure: Edith, Lorina, and Alice Liddell

EAT ME

- Alice goes down a rabbit hole
- Drinks a potion which makes her shrink small
- Then eats cake which makes her stretch tall
- ALICE: “Who in the world am I?... I'm sure I can't be Mabel, for I know all sorts of things,... she knows such a very little!”

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ALICE: “I’ll try if I know all the things I used to know. Let me see:

four times five is twelve,

and four times six is thirteen,

and four times seven is–

oh dear! I shall never get to twenty at that rate! However, the Multiplication Table doesn’t signify: let’s try Geography.”

Alice's Arithmetic

Yes, $4 \cdot 5 = 20$, but 20 looks different expressed in new bases...

BASE	EXPANSION	DIGITS
10 (decimal)	$2 \cdot 10^1 + 0 \cdot 10^0$	20_{10}

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18 (octadecimal)	$1 \cdot 18^1 + 2 \cdot 18^0$	12_{18}

In what base b does $4 \cdot 6 = 13_b$?

Hookahs and Mushrooms

- Alice shrunk again.
- CATERPILLAR: "I've something important to say! Keep your temper."
- ALICE: "Is that all?"
- CATERPILLAR: "No... One side will make you grow taller, and the other side will make you grow shorter... Of the mushroom."

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- The literal translation is **restoration and reduction.**
- De Morgan: "reduce" from universal arithmetic to purely symbolic operations, and, eventually, "restore" meaning.
- Alice too wanted to restore and reduce.
- Keeping her 'temper' means to maintain her correct proportions even when her overall size changes,
- just like similar triangles have common ratios.

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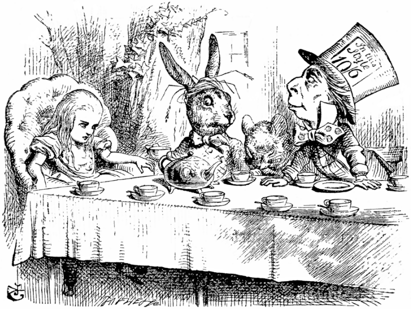
The Mad *t* Party

Figure: Oops! Wrong Party.

- HATTER: “Why is a raven like a writing-desk?”
- ALICE: “I believe I can guess that.”
- HARE: “Then you should say what you mean.”
- ALICE: “At least I mean what I say–that’s the same thing, you know.”
- HATTER: “You might just as well say that ‘I see what I eat’ is the same thing as ‘I eat what I see’!”



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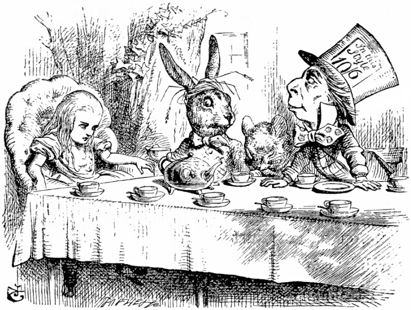

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- HATTER: “It’s always tea-time, and we’ve no time to wash the things.”
- ALICE: “Then you keep moving round?”
- HATTER: “Exactly so.”
- HARE: “Take some more tea.”

- ALICE: “I’ve had nothing yet, so I can’t take more.”
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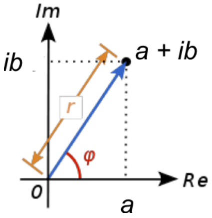
- Likewise, some Victorians rejected imaginary numbers: bi where b is real but $i^2 = -1$.
- Multiplication by a complex number $a + bi = re^{i\varphi}$ scales by r and rotates by an angle of φ.



- In 1843, Sir William Hamilton discovered numbers which represent scaling and rotations in 3D.

- These number are called **quaternions** and are of the form

$$a + bi + cj + dk$$

where a, b, c, d are real, but $i^2 = j^2 = k^2 = ijk = -1$.

- Note: $ij = k \neq -k = ji$, so quaternions don't commute, as in "I get what I like" is not the same as "I like what I get."

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- Previously, Hamilton used three components (the Hatter, Hare, and Doormouse), but could only compute rotations in a plane (stuck moving around the table).

- Hamilton viewed the 4th component as time t (Time had a falling out with the Hatter and was missing from the party).

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Figure: Gimbal Lock

Jabberwocky

'Twas brillig, and the slithy toves
Did gyre and gimble in the wabe:
All mimsy were the borogoves,
And the mome raths outgrabe.

"Beware the Jabberwock, my son!
The jaws that bite, the claws that catch!
Beware the Jubjub bird, and shun
The frumious Bandersnatch!"

He took his vorpal sword in hand:
Long time the manxome foe he sought –
So rested he by the Tumtum tree,
And stood awhile in thought.

And, as in uffish thought he stood,
The Jabberwock, with eyes of flame,
Came whiffling through the tulgey wood,
And burbled as it came!

- How do we know which language a gibberish word is in?
- My friend Dr. John Kerl studied just that...
- He used Markov chains of order 2...

$$P(X_n = x_n | X_1 = x_1, \ldots, X_{n-1} = x_{n-1})$$
$$= P(X_n = x_n | X_{n-2} = x_{n-2}, X_{n-1} = x_{n-1}),$$

  so we assume the chance of getting a letter in a word can be reasonably determined by the previous two letters.
- He fed in word lists of around one-hundred thousand, and stored appropriate probabilities in transition matrices

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He generated the following gibberish words in various languages. Can you tell which languages are being used?

1. churency kingling supprotophated doconic linictoxly stewalorties murine hawkinesses
2. perónimo bolón sanfija morricete esmotorrar bisfato filamberecer estempolí mícleta zarífero senestrosia desalificapio
3. Böservolle techtausfälle Nah wohlassee verschützen Probinus träßcher Postenpland einprückt Bußrfere höhegendeter
4. occlamo domitor nestum inhibeo prohisus equino eribro obvolla exteptor exibro abduco loci equa occasco