Slumdog Millionaire

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Outline

Quartic Formula

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- an orthodox Brahmin (Hindu) and a strict vegetarian

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- He said "An equation for me has no meaning, unless it represents a thought of God,"
- but he also remarked that all religions seemed equally true to him.

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He then solved the quartic equation

$$x^4 + ax^3 + bx^2 + cx + d = 0$$

on his own; one of the roots is given by $x = \ldots$

$$\frac{-a}{4}-\frac{1}{2}\sqrt{\frac{a^2}{4}-\frac{2b}{3}+\frac{2^{\frac{1}{3}}\left(b^2-3ac+12d\right)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4\left(b^2-3ac+12d\right)^3+\left(2b^3-9abc+27c^2+27a^2d-72bd\right)^2}\right)^{\frac{1}{3}}}}$$

$$\frac{-a}{4} - \frac{1}{2}\sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}\left(b^2 - 3ac + 12d\right)}{3\left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4\left(b^2-3ac+12d\right)^3 + \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^2}\right)^{\frac{1}{3}}}}$$

$$+\left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4\left(b^2-3ac+12d\right)^3 + \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd\right)^2}}{54}\right)^{\frac{1}{3}}$$

$$\frac{\displaystyle \frac{-a}{4}-\frac{1}{2}\sqrt{\frac{a^2}{4}-\frac{2b}{3}+\frac{2^{\frac{1}{3}}\left(b^2-3ac+12d\right)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4\left(b^2-3ac+12d\right)^3+\left(2b^3-9abc+27c^2+27a^2d-72bd\right)^2}\right)^{\frac{1}{3}}}}}{\displaystyle +\left(\frac{2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4\left(b^2-3ac+12d\right)^3+\left(2b^3-9abc+27c^2+27a^2d-72bd\right)^2}}{54}\right)^{\frac{1}{3}}}$$

$$-\frac{1}{2}\sqrt{\frac{a^2}{2}-\frac{4b}{3}-\frac{2^{\frac{1}{3}}\left(b^2-3ac+12d\right)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4\left(b^2-3ac+12d\right)^3+\left(2b^3-9abc+27c^2+27a^2d-72bd\right)^2}\right)^{\frac{1}{3}}}}$$

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$$
\begin{aligned}
&\frac{-a}{4}-\frac{1}{2}\sqrt{\frac{a^2}{4}-\frac{2b}{3}+\frac{2^{\frac{1}{3}}\left(b^2-3ac+12d\right)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4\left(b^2-3ac+12d\right)^3+\left(2b^3-9abc+27c^2+27a^2d-72bd\right)^2}\right)^{\frac{1}{3}}}} \\
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&\quad \overline{-\left(\frac{2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4\left(b^2-3ac+12d\right)^3+\left(2b^3-9abc+27c^2+27a^2d-72bd\right)^2}}{54}\right)^{\frac{1}{3}}} \\
&\quad \overline{-\frac{-a^3+4ab-8c}{4\sqrt{\frac{a^2}{4}-\frac{2b}{3}+\frac{2^{\frac{1}{3}}\left(b^2-3ac+12d\right)}{3\left(2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4\left(b^2-3ac+12d\right)^3+\left(2b^3-9abc+27c^2+27a^2d-72bd\right)^2}\right)^{\frac{1}{3}}}+\left(\frac{2b^3-9abc+27c^2+27a^2d-72bd+\sqrt{-4\left(b^2-3ac+12d\right)^3+\left(2b^3-9abc+27c^2+27a^2d-72bd\right)^2}}{54}\right)^{\frac{1}{3}}}}}
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It's not that we just can't find it... we've proven that no such formula exists!

Nested Expressions

In 1904, he was given a scholarship to his hometown college, but he lost it after one year because he neglected all but math.

Figure: Temple in Kumbakonam

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He tried to attend the University of Madras in 1906, but failed every subject on his admittance exam except math.

He became ill again in 1909, was married to a 9 year old girl, and wound up at the *Journal of the Indian Math. Society*:

$$? = \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + \cdots}}}}$$

He posed the above problem, going unsolved for over 6 months.

Note that

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Ramanujan was also a master of continued fractions...

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}$$

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$$\varphi^2 - \varphi - 1 = 0.$$

Thus

$$\varphi = \frac{1 + \sqrt{5}}{2} = 1.61803398\ldots$$

Figure: φ is the Golden Ratio

Ramanujan found a relationship between φ, π, and e:

$$\frac{e^{-2\pi/5}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \cfrac{e^{-8\pi}}{1 + \cdots}}}}$$

Notebooks and Nonsense

"A short uncouth figure... walked in with a frayed notebook under his arm. He was miserably poor... He opened his book and began to explain some of his discoveries... but my knowledge did not permit me to judge whether he talked sense or nonsense." —Ramachandra Rao

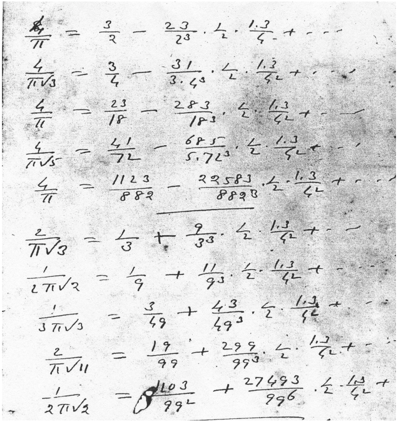
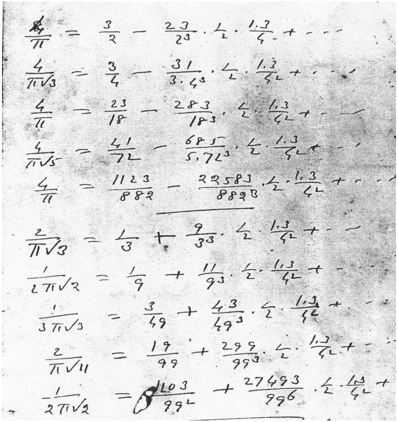


Figure: A Page in the Notebooks

As paper was scare, Ramanujan wrote only his final results on paper.



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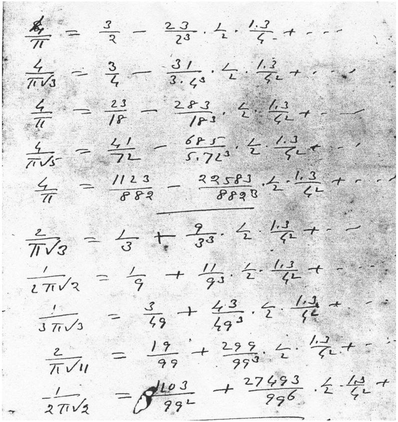

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There are some results which took mathematicians many years after his death to verify.

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Hardy remarked that Ramanujan’s theorems “must be true, because, if they were not true, no one would have the imagination to invent them.”

$$1 + 2x + x^2 \overline{)\,1 \qquad\qquad\qquad\qquad\qquad\qquad}$$

$$\begin{array}{r|l} & 1 \\ \hline 1+2x+x^2 & 1 \\ & -(1+2x+x^2) \\ \hline & \quad -2x-x^2 \end{array}$$

$$
\begin{array}{r l}
 & 1 - 2x \\
1 + 2x + x^2 \,\big) & 1 \\
 & -\underline{(1 + 2x + x^2)} \\
 & \quad -2x - x^2 \\
 & - \quad \underline{(-2x - 4x^2 - 2x^3)} \\
 & \qquad\qquad 3x^2 + 2x^3
\end{array}
$$

$$
\begin{array}{rl}
 & 1 - 2x + 3x^2 \\
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 & \quad -2x - x^2 \\
 & -\underline{(-2x - 4x^2 - 2x^3)} \\
 & \qquad 3x^2 + 2x^3 \\
 & -\underline{(3x^2 + 6x^3 + 3x^4)} \\
 & \qquad\qquad -4x^3 - 3x^4
\end{array}
$$

$$
\begin{array}{r l}
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 & \qquad 3x^2 + 2x^3 \\
 & - \qquad \underline{(3x^2 + 6x^3 + 3x^4)} \\
 & \qquad\quad - 4x^3 - 3x^4 \\
 & - \qquad\quad \underline{(-4x^3 - 8x^4 - 4x^5)} \\
 & \qquad\qquad 5x^4 + 4x^5
\end{array}
$$

Thus

$$1 - 2 + 3 - 4 + \cdots \text{ ``}=\text{''} \frac{1}{1^2 + 2 \cdot 1 + 1} = \frac{1}{4}$$

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so

$$1+2+3+4+\cdots \text{ ``}=\text{''} (1/4)/(-3)=-\frac{1}{12}$$

Hardy helped arrange for Ramanujan to come to England in 1914 so they could collaborate on research.

Unfortunately, the outbreak of World War I made specialty vegetarian food scare. This along with stress made Ramanujan fell ill with tuberculosis and a vitamin deficiency.

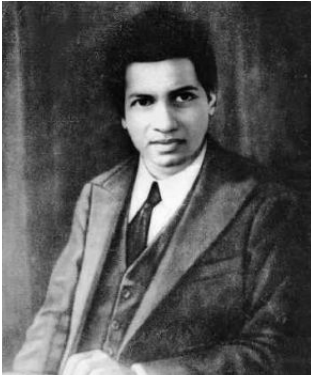

Figure: Photo of Ramanujan

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Hardy later wrote “I remember once going to see [Ramanujan] when he was ill... I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one...

‘No,’ he replied, ‘it is a very interesting number; it is the smallest number expressible as the sum of two [positive] cubes in two different ways’.”

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

The Riemann Zeta Function $\zeta(s)$

For $s > 1$ define

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

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This is THE most important function in mathematics and is **intimately** connected with prime numbers p; e.g.,

$$\zeta(s) = \left(1 - \frac{1}{2^s}\right)^{-1} \left(1 - \frac{1}{3^s}\right)^{-1} \left(1 - \frac{1}{5^s}\right)^{-1} \cdots = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1}$$

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We can extend the definition "smoothly" of $\zeta(s)$ to all complex numbers $s \neq 1$.

Table: Special Values of $\zeta(s)$

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-1	$-1/12$	bosonic string theory

The Riemann Hypothesis: One of six remaining Millennium Problems whose solution will result in a 1,000,000 dollar prize. The problem is prove that all non-trivial zeros of $\zeta(s)$ lie on the vertical line $x = 1/2$ in the complex $z = (x + iy)$-plane.

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Ramanujan may have very well proved this himself had he not died at age 32 in 1920.