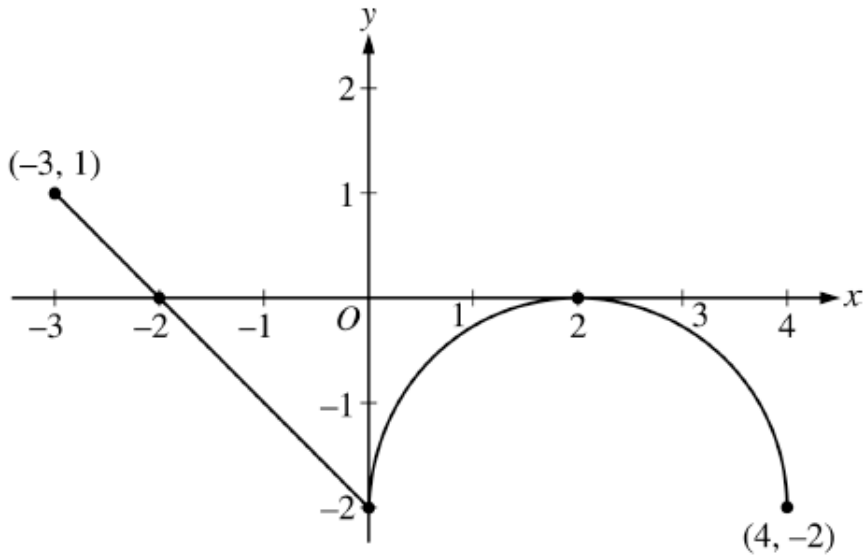
1. Let f be a real-valued function defined on $[-3, 4]$ with $f(0) = 3$. The graph of the derivative f' is shown below.



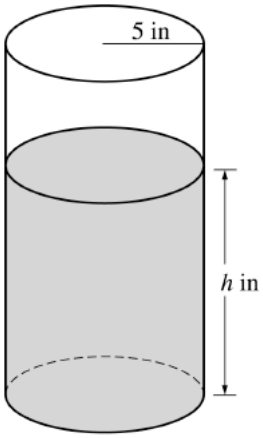
Graph of f'

(a) On what intervals is f increasing? Explain.

(b) Find the x-coordinate of each inflection point of f on the interval $(-3, 4)$.

(c) Find an equation for the tangent line to the graph of f at the point $(0, 3)$

2. Consider the cylindrical coffee pot shown below where h is the depth in inches of the coffee in the pot as a function of time t measured in seconds. Suppose the volume V of the coffee in the pot is changing at a rate of $-5\pi\sqrt{h}$ cubic inches per second.



(a) Show that $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$.

(b) Find h as a function of t given that $h = 17$ at $t = 0$.

(c) When is the coffee pot empty?

3. (a) Let $f(x)$ be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5 - x & \text{for } 3 < x \leq 5. \end{cases}$$

Is f continuous at 3? Explain.

(b) Suppose $g(x)$ is given by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx + 2 & \text{for } 3 < x \leq 5 \end{cases}$$

where m, k are constants. If g is differentiable at $x = 3$, what are the values of k and m?

4. (a) Compute

$$\lim_{x \to 0} \frac{\tan(x^2)}{x}$$

and justify your answer.

(b) Compute

$$\lim_{x \to \infty} x^{1/x}$$

and justify your answer.

5. Consider the circle C of radius 2 centered at the origin.

(a) Find the slopes of the tangent lines to C at the points with y-coordinate -1.

(b) Find the intersection point of the normal lines to these points.