Hyperbolic Geometry, Complex Periods, Stereoscopy, and 4D

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University of Arizona

2/2/11

Outline

M.C. Escher

Circle Limits

Print Gallery

Salvador Dalí

Harmony of the Spheres

Crucifixion

M.C. Escher

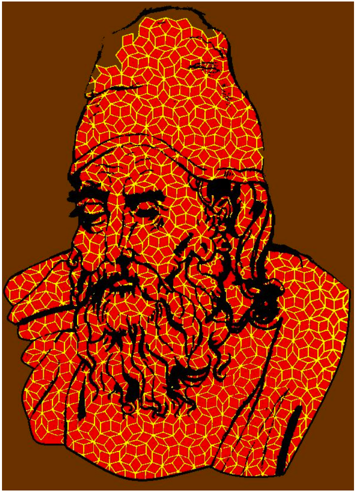
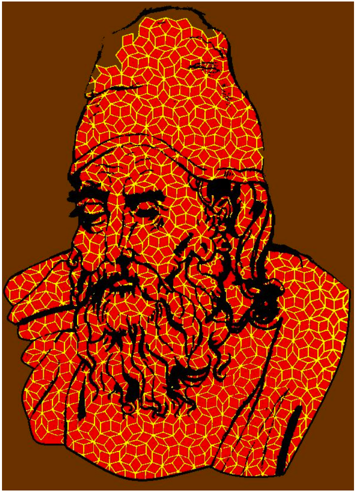


Figure: Euclid of Alexandria

- Wrote *The Elements* in 300 B.C.
- All geometry comes down to 5 “postulates”

  1. There are line segments
  2. There are lines
  3. There are circles
  4. Right angles are congruent
  5. Fishy parallel postulate



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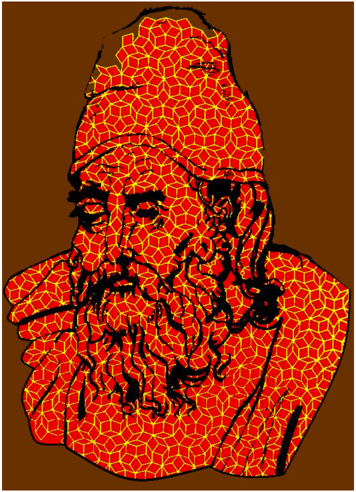

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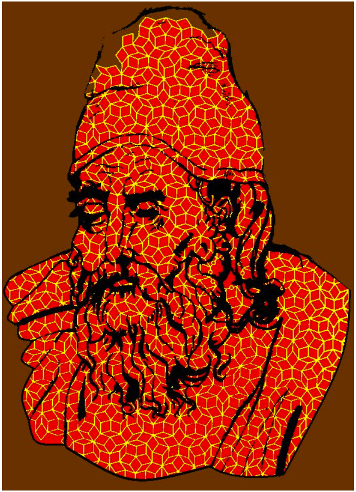

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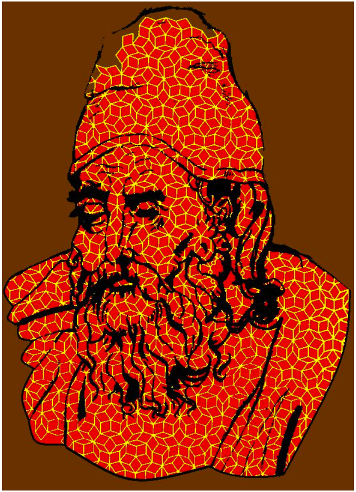

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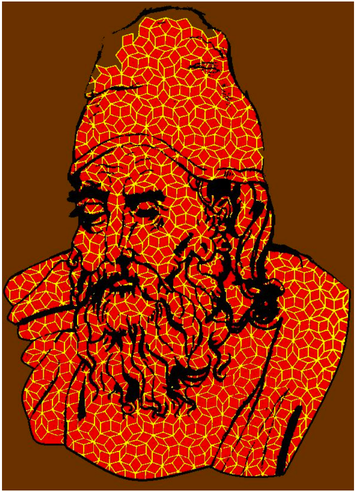

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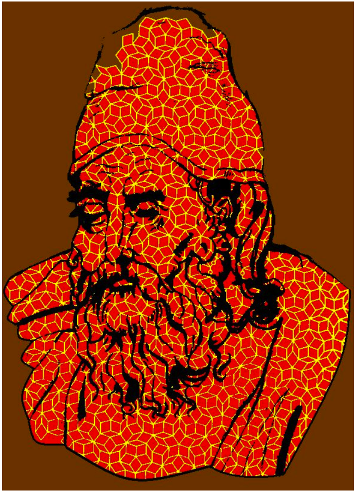

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Euclid's Fifth Postulate

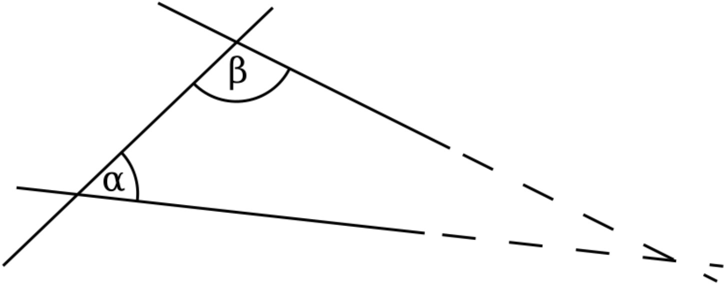

Figure: If $\alpha + \beta < \pi$, then lines will eventually intersect

- Seems like more of a result than an assumption.
- Spent 2,000 yr.s trying to deduce the 5th from 1–4.

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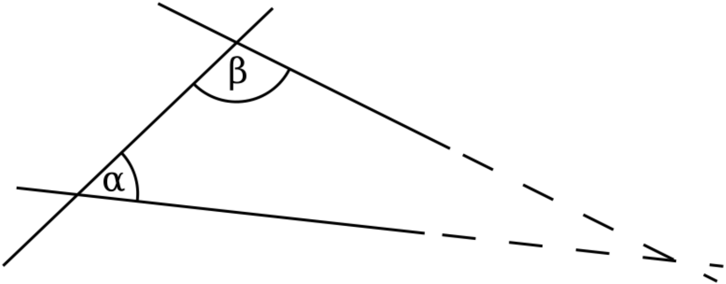

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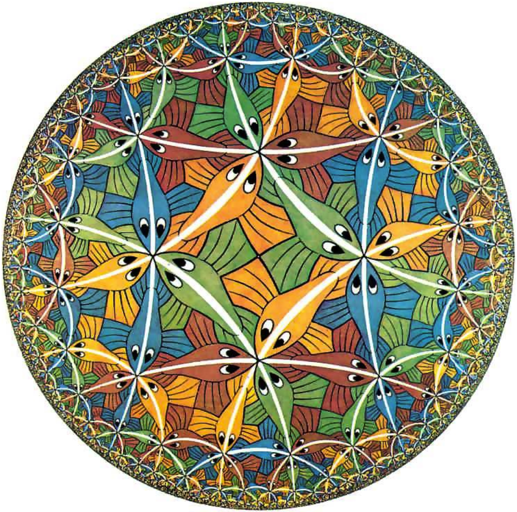


Figure: "Circle Limit III," 1959

In the 1830's, we realized such a proof was impossible since a "new" geometry satisfied 1–4, but not 5.

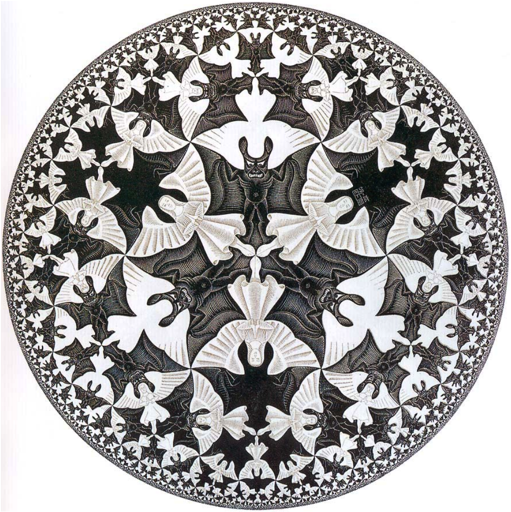


Figure: "Circle Limit IV," 1960

"Out of nothing I have created a strange new universe."
—J. Bolyai commenting on hyperbolic geometry

Figure: “Print Gallery,” 1956

The Droste Effect

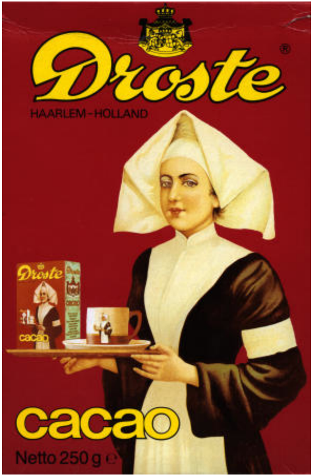
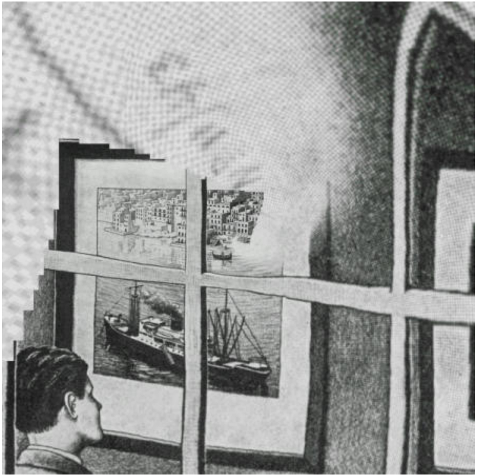


Figure: A self-referential picture

Straightened Print Gallery



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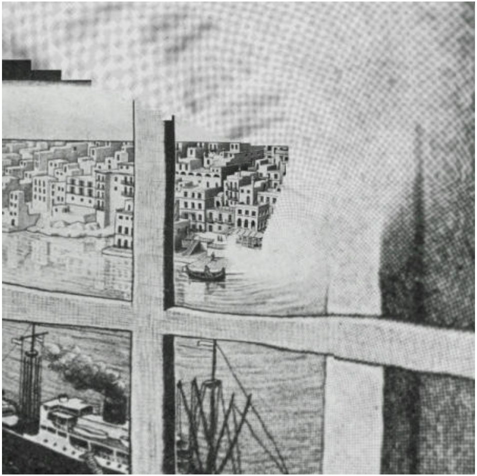

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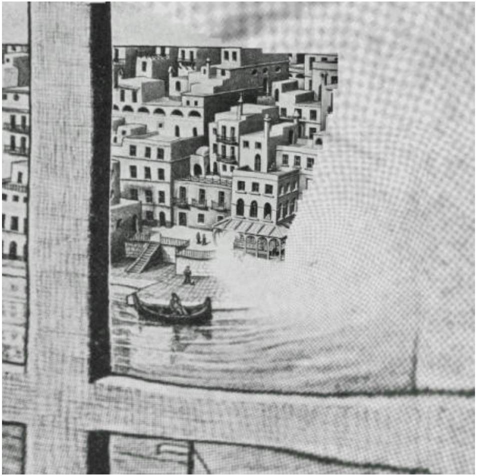

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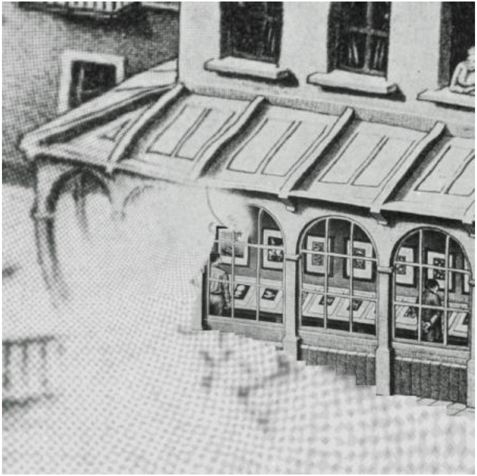

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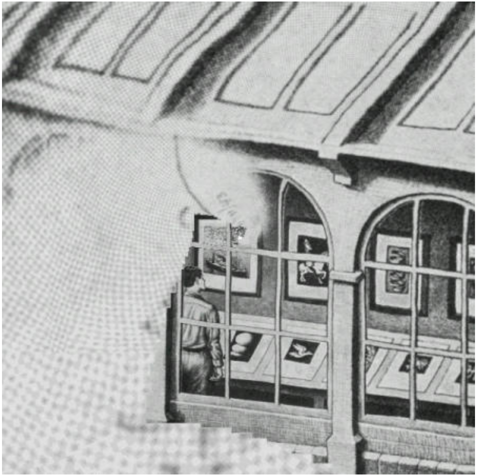

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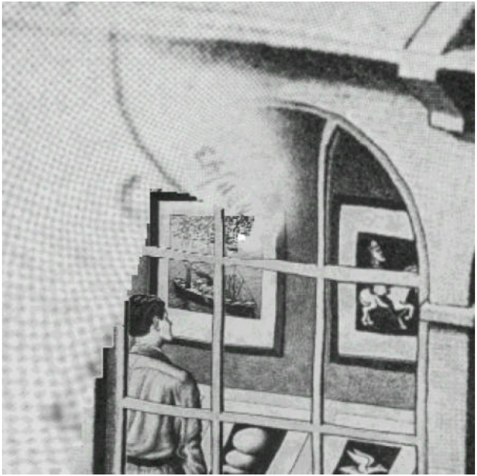
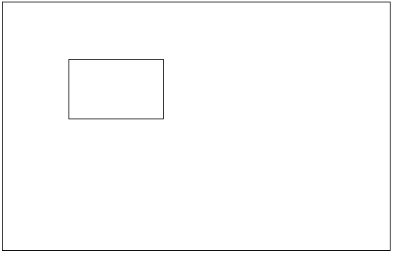


The Math

Every image that contains a copy of itself has a center.



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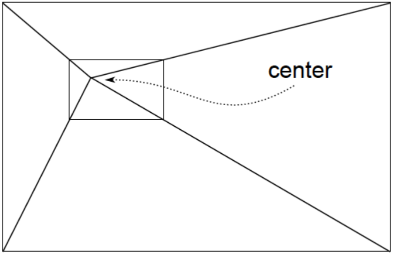
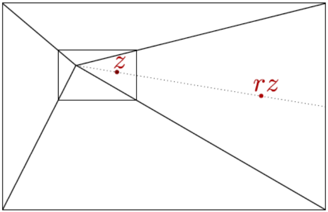


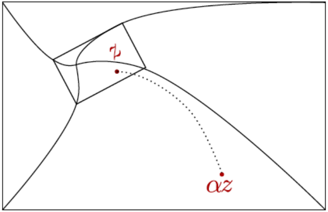
Now place the picture in the complex plane with the center at the origin.

The Math



- The fact that the picture contains a copy of itself just means that it is invariant under multiplication by a scalar r, which we will call the period.

The Math



- The fact that the picture contains a copy of itself just means that it is invariant under multiplication by a scalar r, which we will call the period.
- Escher's twisted picture SHOULD have a *complex period*. That is, it should be a picture that is invariant under both a rotation and a scaling.

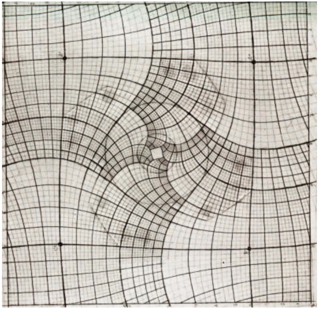


Figure: Escher's grid

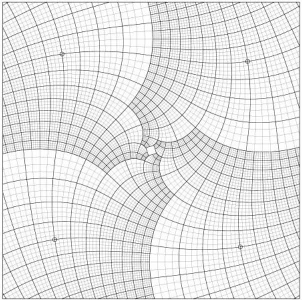
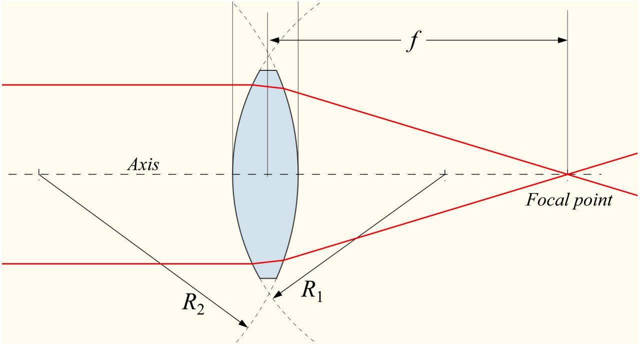


Figure: computer's grid

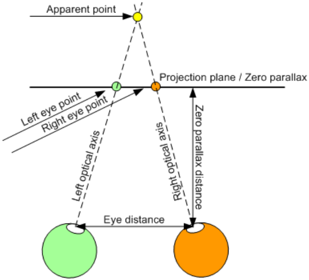
Figure: Loop

Salvador Dalí

Figure: A Holmes-Stereoscope

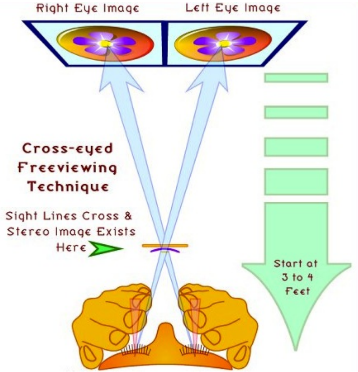


f
Axis
Focal point
R_1
R_2



Apparent point
Projection plane / Zero parallax
Left eye point
Right eye point
Left optical axis
Right optical axis
Zero parallax distance
Eye distance

Another Way of Seeing 3D



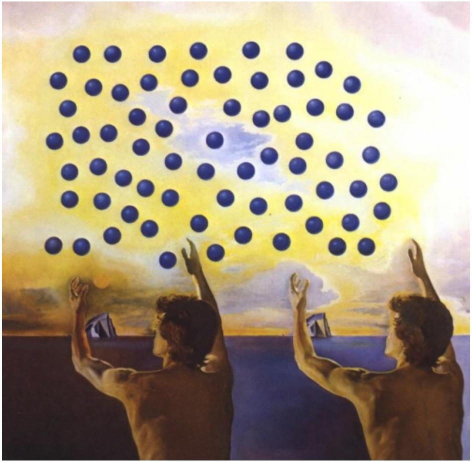


Figure: “The Harmony of the Spheres,” 1978

Figure: Dalí talks with mathematician T. Banchoff in 1975

Figure: “Crucifixion (Corpus Hypercubus),” 1954

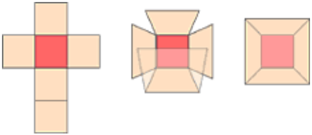


Figure: Folding a cross into a cube

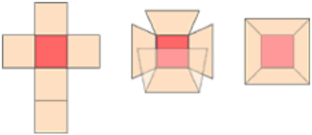


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Figure: Which of these can be folded into a cube?

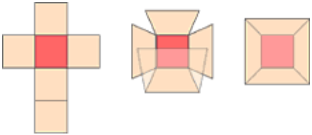


Figure: Folding a cross into a cube

Figure: Which of these can be folded into a cube?

Only a, b, and d. In total, there are 11 unfoldings.

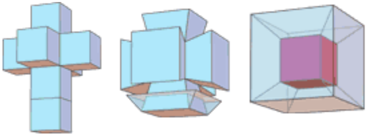
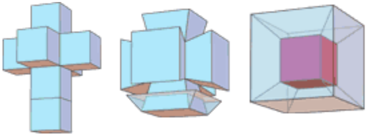


Figure: Folding a hypercross into a hypercube



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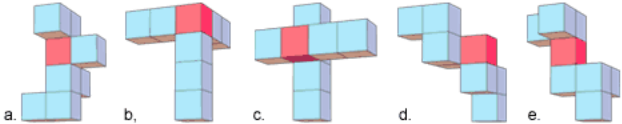

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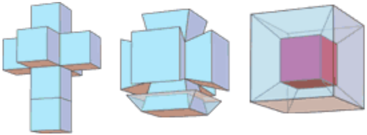

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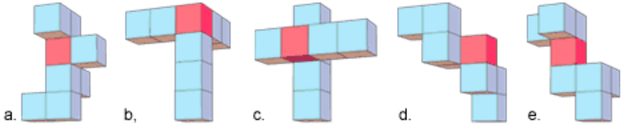

Figure: Which of these can be folded into a hypercube?

Only a, c, and e. In total, there are 261 unfoldings.

Figure: The 4D Hypercube