

## MATH 5B PROBLEM SET #2

**Problem #1:** Let  $l_1$  be the line passing through the points  $(2, 4, 0)$  and  $(3, 1, 1)$  and let  $l_2$  be the line passing through  $(1, 1, 3)$  and  $(0, 5, 1)$ . What is the distance between  $l_1$  and  $l_2$ ?

**Problem #2:** Show that the distance  $d$  between the two parallel planes determined by the equations  $Ax + By + Cz = D_1$  and  $Ax + By + Cz = D_2$  is

$$d = \frac{|D_1 - D_2|}{\sqrt{A^2 + B^2 + C^2}}$$

**Problem #3:** Homer Simpson's favorite equation in cylindrical coordinates is  $(r - 2)^2 + z^2 = 1$ . Sketch the surface it describes to find out why.

*Hint:* What is the intersection of the surface with the half-plane  $\theta = \pi/2$ ?

**Problem #4:** Determine the level curves of the following functions and use that information to (try to) sketch graphs.

- (a)  $f(x, y) = \sqrt{x^2 + y^2}$
- (b)  $g(x, y) = \frac{x}{y}$
- (c)  $h(x, y) = xy$

If you like, you can use a computer mathematics package such as Mathematica or Maple to see if your sketches are accurate.

**Problem #5:** Consider the function

$$f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

- (a) Convert this function to polar coordinates in order to find the level curves. Try to make a rough sketch of the graph of  $f$ .
- (b) Determine  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ , if it exists.
- (c) Based on your answer to part (b), is  $f$  continuous? Explain.