MATH 5B PROBLEM SET #4

Problem #1: Consider a system of two gears, one of radius 2 centered at $(2,0)$ and another of radius 1 centered at $(-1,0)$. The gear of radius 2 rotates counterclockwise at unit angular velocity while the gear of radius 1 rotates clockwise without slipping. Each gear has a peg on its circumference and the two pegs are connected by an elastic band. Initially, the two pegs are at the origin.

(a) Parameterize the curve in the plane traced out by the midpoint P of the elastic band.
(b) Find the velocity and acceleration of P.
(c) Write down an integral representing the distance traveled by the point P in the time it takes for the larger gear to make two revolutions.
(d) Is the curve traced out by P a closed curve (i.e. is it periodic)? If so, what is the period? If not, why not?

Problem #2: Do the following:

(a) Integrate the function $f(x,y) = x+y$ over the region bounded by $x+y=2$ and $y^2-2y-x=0$.

(b) Integrate the function $f(x,y) = y\cos(x^2)$ over the region bounded by the the curves $y=0$, $x=4$, and $x=y^2$. *Caution:* Be careful about order of integration here.

(c) Set up but do not evaluate an integral representing the volume of the solid bounded by the surfaces $z=4x^2+y^2$ and $y^2+z^2=2$.

(d) Integrate $f(x,y,z) = 1-z^2$ over the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$.

Problem #3: Consider the triple integral

$$\int_{-1}^{1}\int_{y^2}^{1}\int_{0}^{1-x} f(x,y,z)dz\,dx\,dy$$

Sketch the region of integration and rewrite the integral in five different ways using different orders of integration.

Problem #4: Do the following:

(a) Evaluate

$$\int_0^2\int_{x/2}^{x/2+1} x^5(2y-x)e^{(2y-x)^2}dy\,dx$$

(b) Find

$$\int\int\int_W (2+x^2+y^2)dV$$

where W is the region inside the sphere $x^2+y^2+z^2=25$ and above the plane $z=3$.

(c) Evaluate

$$\int\int\int_W \sqrt{x^2+y^2+z^2}e^{x^2+y^2+z^2}dV$$

where W is the region bounded by the two spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$, $0 < a < b$.

(d) Evaluate

$$\int\int_D \frac{xy}{y^2 - x^2} dA$$

where D is the region in the first quadrant bounded by the hyperbolas $x^2 - y^2 = 1$, $x^2 - y^2 = 4$ and the ellipses $x^2/4 + y^2 = 1$, $x^2/16 + y^2/4 = 1$.

Hint: Instead of working with the forward transformation $T(u, v) = (x, y)$, focus on the inverse transformation $T^{-1}(x, y) = (u, v)$ and recall that $det(A^{-1}) = \frac{1}{det(A)}$ for an invertible matrix A.

Problem #5: Let B be the ball of radius 3 in $\mathbb{R}^3$ centered at the origin. Use symmetry and geometric arguments to determine the value of $\int\int\int_B (z^3 + 2) dV$. Do not calculate this integral explicitly.