MATH 5B PROBLEM SET #5

Problem #1: Line and Surface Integrals. Do the following:

(a) Let $f(x, y, z) = 3x + xy + z^3$ and let $c(t) = (\cos(4t), \sin(4t), 3t)$, $t \in [0, 2\pi]$. Find $\int_c f\, ds$.

(b) Let $c(t)$ be a path and T the unit tangent vector. What is $\int_c T \cdot d\mathbf{s}$?

(c) Let $c(t) = (e^{2t} \cos(3t), e^{2t} \sin(3t))$, $t \in [0, 2\pi]$. Find $\displaystyle\int_c \frac{x\, dx + y\, dy}{(x^2 + y^2)^{3/2}}$.

(d) Set up but do not evaluate an integral that represents the surface area of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

(e) Let D be the dome shaped region bounded by $z = 8 - 2x^2 - 2y^2$ and the xy-plane and let S be the boundary surface of D. Let $f(x, y, z) = x^2 + y^2 + 3(z - 2)^2$ and let $F = \nabla f$. Calculate the flux of F through S.

Problem #2: Use Green's Theorem. Do the following:

(a) Evaluate

$$\oint_C (x^2 - y^2)\, dx + (x^2 + y^2)\, dy$$

where C is perimeter of the rectangle with vertices $(0, 0)$, $(2, 0)$, $(0, 1)$, and $(2, 1)$.

(b) Show that for any closed curve C in the plane

$$\oint_C 3x^2 y\, dx + x^3\, dy = 0$$

(c) Sketch the curve given parametrically by $c(t) = (1 - t^2, t^3 - t)$ and find the area enclosed by it.

Problem #3: Use Divergence or Stokes's Theorem. Do the following:

(a) Let S be the surface defined by $x^2 + y^2 + 5z = 1$, $z \geq 0$. and let $F(x, y, z) = (xz, yz, x^2 + y^2)$. Verify Stokes's theorem for this surface and vector field.

(b) Let S be the surface defined by $y = 10 - x^2 - z^2$, $y \geq 1$ and let $F(x, y, z) = (2xyz + 5z, e^x \cos(yz), x^2 y)$. Find $\iint_S \nabla \times F \cdot d\mathbf{S}$.

(c) Let S be the surface defined by $z = e^{1 - x^2 - y^2}$, $z \geq 1$ and let $F(x, y, z) = (x, y, 2 - 2z)$. Calculate $\iint_S F \cdot d\mathbf{S}$.