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Mini-Workshop: Artin Groups meet Triangulated Categories

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Abstract. Artin and Coxeter groups are naturally occurring generalisations of the braid and symmetric groups respectively. However, unlike for Coxeter groups, many basic group theoretic questions remain unanswered for general Artin groups – most notably the $K(\pi,1)$-*conjecture for Artin groups* remains open except for certain special families of Artin groups. Recently, Artin groups have also appeared as groups acting on triangulated categories, where the associated spaces of Bridgeland's stability conditions provide new realisations of the corresponding $K(\pi,1)$ spaces. The aim of the workshop is to bring together experts and early career researchers from two seemingly different areas of research: (i) geometric and combinatorial group theory and topology, and (ii) triangulated categories and stability conditions, to explore their intersection via the $K(\pi,1)$-conjecture.

Mathematics Subject Classification (2020): 20F36, 18G80.

Introduction by the Organizers

Artin and Coxeter groups are ubiquitous in mathematics and have facilitated applications in geometry and topology; such as Davis' construction of a counterexample to the conjecture that all aspherical manifolds have Euclidean spaces as universal covers [Dav83] and Agol's proof of the virtual Haken conjecture [Ago13]. However, unlike for Coxeter groups, it is surprising how many basic group theoretic questions remain unanswered for general Artin groups. For example, it is unknown whether Artin groups are torsion-free, and whether they a have solvable word problem (a broader overview can be found in [McC17]). The most prominent conjecture is the $K(\pi,1)$-*conjecture for Artin groups*, which is only proven for certain special families of Artin groups, notably the finite (spherical) types [Del72], FC types [CD95], and more recently, the affine types [PS21] and a new hyperbolic-type family [HH23].

On the other hand, there has been a trend towards studying Coxeter and Artin groups via their actions on triangulated categories, whose associated complex manifolds of Bridgeland's stability conditions provide new realisations of the corresponding $K(\pi, 1)$ spaces [QW18, AW22]. This opens up an untapped wealth of tools, which – when combined with the combinatorial and geometric tools used to study Artin groups – could lead to powerful new insights in the study of Artin and Coxeter groups.

The aim of this workshop was to bring together experts and early career researchers in two areas of research – geometric and combinatorial group theory, and the theory of stability conditions – to discuss methods and problems arising from two different point of views surrounding Artin groups. As preparation for the workshop, the participants received two sets of notes that aimed to establish a common ground for further discussions. Each set of notes was roughly 20-30 pages long, one on the geometric group theory's view on Artin groups by the first-named organiser, and one on Bridgeland's stability conditions by the second-named organiser.

On Monday, two experts of the respective areas, Jon McCammond and Anthony Licata, gave two lectures each as a gentle introduction to the respective research areas. The day was concluded with an exercise session on both topics, that led to a lively discussion. A short introduction of all the participants (ice breaker) was carried out after dinner.

Throughout the remainder of the week, a total of 11 research talks were presented by the participants. These included in particular presentations by graduate students as well as by our online participant. On Tuesday, a SAGE program that calculates actions of braid groups and (semi)stable objects with respect to a given Bridgeland's stability condition were presented by Bapat. On Thursday, an open problem session was held where we compiled a list of open questions, varying in breadth, suggested by the participants.

The (inter-related) topics presented and discussed throughout the week are summarised as follows:

- **Singularities, polynomials and non-crossing partitions:** Shimpi presented a talk on triangulated categories associated to resolutions of singularities, together with the appearance of Artin group actions on them and their relation to the $K(\pi, 1)$-conjecture. Keating also presented a natural construction of actions of Artin groups on triangulated categories via singularity theory, using plumbing of spheres and spherical (Dehn) twists. McCammond presented an explicit relation between hyperplane complements and configuration spaces via polynomials, and showed that the orthoscheme metric on the complex of non-crossing partitions can be naturally induced from the space of polynomials. Relationship between non-crossing partitions and (specific types of) stability conditions in the type A case were also discussed during the exercise session. Related to this, Bianchi presented results on the homology of configuration spaces of surfaces.

- **Hyperplane complements and their universal covers:** Certain subspaces of hyperplane complements were presented by Wemyss as naturally occurring objects in the theory of flops, some of which are not associated to Coxeter groups but are indistinguishable from this point of view (any reasonable proof of the $K(\pi, 1)$-conjecture will prove that they are all $K(\pi, 1)$-spaces). Dell also talked about subspaces of hyperplane complements in relation to the non-simply-laced type Artin groups, where a similar relation on the space of stability conditions occurs via the submanifold of fusion-equivariant stability conditions (which conjecturally covers the non-simply-laced type hyperplane complements). Algebraic geometric consequences from the understanding of stability conditions were also presented in both Wemyss' and Dell's talks.
- **Actions of Artin groups on metric spaces:** Various simplicial and cube complexes upon which Artin groups act were introduced in talks of Mastrocola, Corrigan, Huang and McCammond. Interesting metrics were put on these spaces, and the geometric properties of the action were utilised in proving group-theoretical properties of Artin groups. On the other hand, Deopurkar discussed stability conditions as metrics on triangulated categories, which prompts the possibility of using metrics on triangulated categories to study groups acting on them. In the nearby world of Coxeter groups, Schwer introduced a meta-complex, which encoded the famous *isomorphism problem* for Coxeter groups.

References

[Ago13] Ian Agol. The virtual Haken conjecture. *Doc. Math.*, 18:1045–1087, 2013. With an appendix by Ian Agol, Daniel Groves, and Jason Manning.

[AW22] Jenny August and Michael Wemyss. Stability conditions for contraction algebras. *Forum Math. Sigma*, 10:Paper No. e73, 20, 2022.

[CD95] Ruth Charney and Michael Davis. The K(π,1)-problem for hyperplane complements associated to infinite reflection groups. *Journal of the American Mathematical Society*, 8(3):597–627, 1995.

[Dav83] Michael W. Davis. Groups generated by reflections and aspherical manifolds not covered by Euclidean space. *Ann. of Math. (2)*, 117(2):293–324, 1983.

[Del72] P. Deligne. Les immeubles des groupes de tresses généralisés. *Inventiones Mathematicae*, 17:273–302, 1972.

[HH23] Thomas Haettel and Jingyin Huang. New Garside structures and applications to Artin groups, 2023.

[McC17] Jon McCammond. The mysterious geometry of Artin groups. *Winter Braids Lect. Notes*, 4(Winter Braids VII (Caen, 2017)):Exp. No. 1, 30, 2017.

[PS21] Giovanni Paolini and Mario Salvetti. Proof of the $K(\pi, 1)$ conjecture for affine Artin groups. *Invent. Math.*, 224(2):487–572, 2021.

[QW18] Yu Qiu and Jon Woolf. Contractible stability spaces and faithful braid group actions. *Geom. Topol.*, 22(6):3701–3760, 2018.

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Mini-Workshop: Artin Groups meet Triangulated Categories

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Minicourse: Artin Groups and Triangulated Categories

Tony Licata

This short course consisted of two one-hour lectures. The first lecture concerned generalities of group actions on triangulated categories. We introduced strong and strict actions and briefly discussed how different appearances of a group action (combinatorial and intrinsic) give rise naturally to weak or strict actions. We mentioned, but did not fully explain, that the weak actions of Artin groups which are the topic of the second lecture can be made strict. We also discussed a few examples of categorical structures (e.g. 2-character theory), which are seen by strict actions but not by weak actions. We discussed the octahedral axiom for triangulated categories and its relation to planar topology. We also introduced the notion of a metric on a triangulated category, and explained how metrics allow one to study triangulated equivalences via tools coming from dynamics.

The second talk concerned categorical actions of Artin groups, focusing on the fundamental "geometric 2-representation" of an Artin group. If (W, S) is a Coxeter system with associated Artin group A_W, then the geometric 2-representation of A_W is a triangulated category T_W, generated by a set of $|S|$ 2-spherical objects. The spherical twists of these objects define a categorical action of A_W on T_W. (In the lectures, we only defined this category when W is simply-laced, though the construction can be generalised to other types.)

The fundamental conjecture concerning the action of A_W on T_W is the following.

Conjecture: The action of A_W on T_W is faithful. More precisely, if $\beta \in A_W$ acts by (a functor isomorphic to) the identity functor on T_W, then $\beta = 1 \in A_W$.

The conjecture has been established in special cases, but remains open in general. A corollary of the conjecture is a solution to the word-problem in A_W.

The latter half of the second lecture concerned the moduli space of stability conditions on T_W, denoted here by $Stab(T_W)$. We explained that a connected component $Stab^*(T_W) \subset Stab(T_W)$ is a covering space of a space Y_W, with $\pi_1(Y_W) \cong A_W$. The fundamental conjecture regarding this space is:

Conjecture: The space $Stab(T_W)$ is contractible.

Again, this is known in examples but open in general. Proving this conjecture would solve many of the open problems about Artin groups.

References

[1] A. Ikeda, *Stability conditions for preprojective algebras and root systems of Kac-Moody Lie algebras*, arXiv:1402.1392

[2] T. Bridgeland, *Stability conditions on triangulated categories*, Annals of Mathematics, 166 (2007), 317–345

[3] B. Elias and G. Williamson, *Diagrammatics for Coxeter groups and their braid groups* Quantum Topol. 8 (2017), no. 3, pp. 413–457

Minicourse: Artin Groups

Jon McCammond

Artin groups are derived from Coxeter groups and both classes of groups are succinctly defined by group presentations of a particularly simple form. These simple presentations, however, obscure the fact that Coxeter groups are well understood, while Artin groups remain fundamentally mysterious, outside of special classes of examples and select portions of the groups.

Artin groups are of interest due to their close connections to other parts of mathematics: (1) braid groups and polynomials (Hurwitz, Artin), (2) Coxeter groups, buildings and Lie theory (Coxeter, Tits, Bourbaki), (3) singularity theory (Arnold, Brieskorn, Deligne, Looijenga), (4) hyperplane arrangements (Orlik, Solomon, Terao, Salvetti), (5) finite complex reflection/braid groups (Broué, Malle, Rouquier, Bessis), (6) noncrossing combinatorics (Reiner, Reading, Chapoton, Stump, Williams), (7) exceptional sequences in hereditary algebras (Hubery, Krause, Thomas), (8) extended affine Artin groups (Saito, Baumeister), and (9) triangulated categories and stability conditions (Bridgeland, Licata, Wemyss). The first three topics are the original source of these groups as an object of study.

The natural action of $\textsc{Sym}_n$ on $\mathbb{C}^n$ is free once we remove the "braid arrangement" of hyperplanes where two coordinates are equal, and the fundamental group of the quotient of the complex hyperplane complement by the free $\textsc{Sym}_n$ action is the braid group. For a more general Artin group, the symmetric group action on $\mathbb{C}^n$ is replaced with the faithful linear action, introduced by Jacques Tits, of the corresponding Coxeter group on a (complexified) metric vector space. The contragradient version of this representation preserves a union of (real) simplicial cones called the Tits' cone. The Coxeter group acts freely on the complexified Tits' cone once the complexified hyperplanes fixed by reflections are removed, and the fundamental group of the quotient by this free action is the corresponding Artin group. The cell structure dual to the Tits' cone is the Davis complex, which supports a nonpositively-curved piecewise Euclidean metric where the cells are polytopes that are generalized versions of permutahedra. For the complexified hyperplane complement, there is a similar cell structure, called the Salvetti complex, which is an oriented version of the Davis complex built out of oriented W-permutahedra. See [1] and the references it cites for details.

Unfortunately, the local metric geometry of the Salvetti complex is not nonpositively curved except in extremely simple cases. As an alternative, there is a dual Garside construction for some classes of Artin groups which appear to have better metric properties, at least in the case of the braid groups, and this is the topic of the Friday research talk.

References

[1] J. McCammond, *The mysterious geometry of Artin groups*, Winter Braids Lecture Notes **4** (2017), 30 pages.

The geometry of stability conditions

Anand Deopurkar
(joint work with Asilata Bapat, Anthony Licata)

Stability conditions bring triangulated categories into the realm of geometry in more than one way. We recall stability conditions and explain a few ways of thinking about them in geometric terms, particularly for the 2-Calabi–Yau (CY) categories associated to quivers defined in the lectures of Anthony Licata (see [2, §2.3]).

Let $\mathcal{C}$ be a triangulated category. A *(Bridgeland) stability condition* on $\mathcal{C}$ consists of a *central charge*, namely a $\mathbf{Z}$-linear map $Z\colon K(\mathcal{C}) \to \mathbf{C}$, and a *slicing*, namely a collection of abelian subcategories $P_\phi \subset \mathcal{C}$ indexed by real numbers ϕ, satisfying a number of conditions (see [3]). The objects of P_ϕ are called semi-stable of phase ϕ. One of the conditions in the definition ensures that every object $x \in \mathcal{C}$ admits a unique filtration, called the *Harder–Narasimhan (HN) filtration*, whose factors are semi-stable and appear in the order of descending phase. A rough analogy is to think of an object of $\mathcal{C}$ as an audio wave, the semi-stable objects as waves of pure frequency, and the decomposition of an object into semi-stable factors as the decomposition of an audio wave into pure frequencies. (This analogy, however, ignores that the HN decomposition is ordered).

A stability condition gives multiple measures of complexity of objects and morphisms. Associated to an object x is its *spread*, namely the difference between the highest and the lowest phases of the semi-stable objects in its HN filtration. The spread is a measure of homological complexity of the object.

Also associated to an object x is its *mass*, defined as follows. The mass of a semi-stable object is the absolute value of its central charge; the mass of an arbitrary object is the sum of the masses of its semi-stable factors. The mass gives another measure of complexity of the object, which is of a different flavour than the spread. The mass also allows us to define a metric on the category by declaring the length of a morphism to be the mass of its cone. With this metric, we can think of a sequence of composable morphisms in the category as a path. The sequence given by a HN filtration turns out to be a *geodesic* path. For 2-CY categories of quivers, we prove that the metric thus induced by a stability condition in fact determines the stability condition ([1, §6.1]). So, we can think of stability conditions as particular kinds of metrics on the category. It will be fantastic to understand exactly which metrics arise from stability conditions.

Thinking of stability conditions as metrics allows us to transfer ideas from metric geometry to the study of triangulated categories. One such instance is a compactification of a stability manifold inspired by Thurston's compactification of Teichmüller space ([1]).

Let $\mathcal{C}$ be the 2-CY category associated to the A_n quiver. Then the Grothendieck group $K(\mathcal{C})$ is the root system of type A_n, which we can take to be the span of the vectors $e_i - e_j$ in $\mathbf{R}^{n+1}$. In this case, there is a beautiful geometric description of stability conditions in terms of configuration of $n+1$ points on the complex plane. Fix such a configuration $\{x_0, \ldots, x_n\} \subset \mathbf{C}$. We define the central charge by $Z(e_i - e_j) = x_i - x_j$. To define the slicing, recall that Khovanov–Seidel give a recipe to construct objects of $\mathcal{C}$ from arcs joining two marked points [4, §4]. We declare the objects represented by the straight line segments to be semi-stable. This turns out to indeed give a stability condition (see [5]). Let x be an object represented by an arc γ. In this stability condition, the HN factors of x correspond to the straight line segment pieces of γ when it is "pulled tight" around the marked points. This geometric description of the HN factors implies many non-trivial properties of the structure of HN filtrations of objects in $\mathcal{C}$. A fundamental question is to understand these properties using pure homological algebra and generalise them to a broader class of categories.

References

[1] Asilata Bapat, Anand Deopurkar, and Anthony M. Licata. A Thurston compactification of the space of stability conditions. *Pre-print*, 2020. http://arxiv.org/abs/2011.07908v1.

[2] Asilata Bapat, Anand Deopurkar, and Anthony M. Licata. Spherical objects and stability conditions on 2-Calabi-Yau quiver categories. *Math. Z.*, 303(1):22, 2023. Id/No 13.

[3] Tom Bridgeland. Stability conditions on triangulated categories. *Ann. of Math. (2)*, 166(2):317–345, 2007.

[4] Mikhail Khovanov and Paul Seidel. Quivers, Floer cohomology, and braid group actions. *J. Amer. Math. Soc.*, 15(1):203–271, 2002.

[5] R. P. Thomas. Stability conditions and the braid group. *Commun. Anal. Geom.*, 14(1):135–161, 2006.

Geometers' twisted retelling of the $K(\pi, 1)$ story

Parth Shimpi

The central objects of the workshop, namely fundamental groups of complexified hyperplane complements, appear as key players in the age-old problem of birational classification in algebraic geometry where they naturally act on triangulated categories associated to reasonable algebraic singularities. The interface between geometric group theory and homological algebraic geometry thus created promises advancements of understanding on both sides. The talk sketches some of these ideas and outlines the progress that has been made towards understanding Artin groups from the perspective of the homological minimal model programme.

1. The categories and invariants

We direct the reader to [12] for an excellent survey of the geometric context and related problems, a part of which is explained below.

Setup 1. Our geometric setup arises from an algebraic variety Y of dimension $d \leq 3$ with a rational singularity $\mathbf{0} \in Y$, and a crepant projective birational morphism $f : X \to Y$ where X has appropriately controlled singularities. Such singular varieties Y occur in families labelled by Dynkin diagrams Δ of type A, D or E. The exceptional fiber $f^{-1}(\mathbf{0})$ is known to be a tree of rational curves which can in turn be indexed by a subset of vertices $J \subset \Delta$. We write $\hat{\Delta} \supset \Delta$ for the corresponding Dynkin diagram of affine type, and $\hat{J} \subset \hat{\Delta}$ for the subset containing J and the extended (special) vertex of $\hat{\Delta}$.

In this case we can define two subcategories of $D^b\mathrm{Coh}X$, namely

$$\mathcal{C} = \{\mathcal{F} \in D^b\mathrm{Coh}X \mid Rf_*(\mathcal{F}) = 0\},$$
$$\mathcal{D} = \{\mathcal{F} \in D^b\mathrm{Coh}X \mid \mathrm{Supp}\mathcal{F} \subset f^{-1}(\mathbf{0})\}.$$

The latter category will witness phenomena of 'affine' type, while $\mathcal{C} \subset \mathcal{D}$ will provide a model for 'finite' type behaviour. To illustrate this, we note that the Grothendieck group $K_0\mathcal{C}$ has a basis indexed by J while $K_0\mathcal{D}$ has a basis indexed by $\hat{J}$. The Grothendieck groups can thus be identified with (restricted) root lattices, inducing hyperplane arrangements $\mathcal{H}, \hat{\mathcal{H}}$ in the respective dual Euclidean space $E, \hat{E}$ as described in [8]. In particular if $J = \Delta$, then we recover the Tits cones of finite-type and affine Coxeter groups.

We remark that these hyperplane arrangements have a variety of homological interpretations in relation to the perverse t-structure [10] on the derived category, for example as the silting fan [11] or as the heart fan [4].

2. The group action

For the hyperplane arrangements arising from either category, the tautological action of the fundamental group of the complexified hyperplane complement can be lifted to an action on the derived category itself.

Theorem 2. *There is a group homomorphism* $\varphi : \pi_1(E \setminus \mathcal{H}) \to \mathrm{Aut}(\mathcal{C})$ *(resp.* $\pi_1(\hat{E} \setminus \hat{\mathcal{H}}) \to \mathrm{Aut}(\mathcal{D})$*) such that the monodromy around a root hyperplane corresponds to the mutation autoequivalences defined in* [13]*. Furthermore, the associated Bridgeland stability manifold has a connected component* $\mathrm{Stab}^\circ(\mathcal{C})$ *(resp.* $\mathrm{Stab}^\circ(\mathcal{D})$*) that is a regular covering space of* $E \setminus \mathcal{H}$ *(resp.* $\hat{E} \setminus \hat{\mathcal{H}}$*) with deck group* $\mathrm{img}(\varphi)$*.*

Remark 3. When $f : X \to Y$ is the minimal resolution of a Kleinian singularity or a 3-fold flopping contraction, the mutation autoequivalences can be interpreted geometrically as coming from spherical twist functors [9] and Bridgeland–Chen flop functors respectively.

3. The questions

A few questions naturally emerge in the above setup.

(1) Is the covering map in theorem 2 the universal cover, i.e. does φ describe a faithful action?
(2) Is the stability manifold connected, i.e. are there no stability conditions outside the connected component appearing in theorem 2?
(3) Is the connected component $\mathrm{Stab}^\circ(\mathcal{C})$ (resp. $\mathrm{Stab}^\circ(\mathcal{D})$) contractible, i.e. is the hyperplane complement $E \setminus \mathcal{H}$ (resp. $\hat{E} \setminus \hat{\mathcal{H}}$) a $K(\pi, 1)$ space?

To answer these, one might seek a complete classification of spherical objects and t-structures on the categories. This is the approach taken in [5] to provide a positive answer for all three questions in relation to category $\mathcal{C}$. In fact there have been multiple successful approaches for answering these questions for the category $\mathcal{C}$. For instance [3], [6] exploit the existence of normal forms in the fundamental group to exhibit faithfulness of the action; [2] study the interaction between spherical objects and stability conditions to exhibit connectedness of the stability manifold; and [1] show that the category satisfies a certain homological property called 'silting discreteness' which guarantees the contractibility of the stability manifold.

Each one of these approaches exploits some aspect of the 'finite-type' nature of $\mathcal{C}$, and fails to generalise to $\mathcal{D}$. Indeed all the questions for $\mathcal{D}$ remain open, except for the minimal resolution of an A_n surface singularity where [7] use an explicit understanding of coherent sheaves and spherical objects to show that the stability manifold $\mathrm{Stab}(\mathcal{D})$ is connected and the action given by φ is faithful.

References

[1] J. August, M. Wemyss, *Stability conditions for contraction algebras*, Forum Math. Sigma **10** (2022), Paper No. e73, 20 pp.

[2] A. Bapat, A. Deopurkar, A. Licata, *Spherical objects and stability conditions on 2-Calabi–Yau quiver categories*, Mathematische Zeitschrift **303** (2023), 13.

[3] C. Brav, H. Thomas, *Braid groups and Kleinian singularities*, Mathematische Annalen **351** (2009), 1005–1017.

[4] N. Broomhead, D. Pauksztello, D. Ploog, J. Woolf, *The heart fan of an abelian category*, arXiv:2310.02844 (2023).

[5] W. Hara, M. Wemyss, *Spherical objects in dimension two and three*, to appear JEMS.

[6] Y. Hirano, M. Wemyss, *Faithful actions from hyperplane arrangements*, Geom. & Topol. **22** (2018), no. 6, 3395–3433.

[7] A. Ishii, K. Ueda, H. Uehara, *Stability conditions on A_n-singularities*, Journal of differential geometry **84** (2010), 87–126.

[8] O. Iyama, M. Wemyss, *Tits cones intersections and applications*, in preparation.

[9] P. Seidel, R. Thomas, *Braid group actions on derived categories of coherent sheaves*, Duke Math. J. **108(1)** (2001), 37–108.

[10] M. Van den Bergh, *Three-dimensional flops and noncommutative rings*, Duke Math. J. **122(3)** (2004), 423–455.

[11] O. van Garderen, *Stability over cDV singularities and other complete local rings*, Advanced Studies in Pure Mathematics **88** (2023), 461-489.

[12] M. Wemyss, *A lockdown survey on cDV singularities*, Adv. Stud. Pure Math. **88** (2023), 47–93.

[13] M. Wemyss, *Flops and clusters in the homological minimal model programme*, Invent. Math. **211** (2018), no. 2, 435–521.

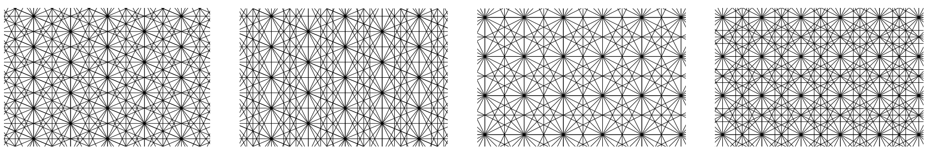
The combinatorics of Tits cones, and related questions

Michael Wemyss

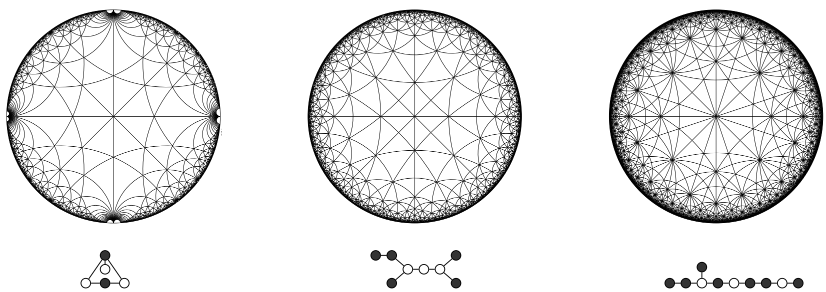
This is an overview talk of generalisations of Coxeter arrangements discovered in [IW]. In algebraic geometry, these arise when studying stability conditions of either partial resolutions of Kleinian singularities, or 3-fold flopping contractions [HW], but from the viewpoint of this talk, where the arrangements came from is not relevant. The point is that from a categorical perspective, these arrangements can't be distinguished from their Coxeter cousins. So, if the stability condition/categorical machine is going to prove the $K(\pi, 1)$-conjecture, almost certainly it will also have to prove that these new arrangements are also $K(\pi, 1)$. From this, various consequences emerge.

The input is a subset of nodes, denoted J, inside a Coxeter graph Δ. Given this data, we can perform a certain intersection inside the Tits cone, and from this obtain a new hyperplane arrangement. This arrangement depends on both J and Δ, but it is not Coxeter in general. Its chambers and walls are, however, labelled by 'Coxeter data', and so many of the features of Coxeter theory remain.

Some special cases which are particularly visually pleasing are those when J is a three-element subset of either (a) an affine ADE diagram, or (b) a hyperbolic diagram. Case (a) is studied in [IW], and examples include



The hyperbolic case (b) is studied by Lewis [L], and examples include



References

[HW] Y. Hirano and M. Wemyss, *Stability conditions for 3-fold flops*, Duke Math. J. **172** (2023), no. 16, 3105–3173.

[IW] O. Iyama and M. Wemyss, *Tits cone intersections and applications*, preprint.

[L] S. Lewis, *Real variations of stability conditions in the hyperbolic setting*, PhD thesis, University of Glasgow, 2024.

The 2-complete Artin complex

Jill Mastrocola

Let Γ be a finite simplicial graph with edges labeled by integers greater than or equal to 2. The Artin group A_Γ is the group with presentation

$$\langle s_1, \ldots, s_n \mid \underbrace{s_i s_j s_i \ldots}_{m_{ij}} = \underbrace{s_j s_i s_j \ldots}_{m_{ij}} \rangle$$

where $s_i \in V(\Gamma)$ and m_{ij} is the label of the edge between s_i and s_j. If there is no edge between s_i and s_j, then those generators have no relation.

For $T \subseteq V(\Gamma)$, the subgroup of A_Γ generated by T is called a *standard parabolic subgroup*, denoted A_T. It is a theorem of Van der Lek that $A_T \cong A_{\Gamma'}$ for Γ' the subgraph of Γ spanned by T [6]. In general, a *parabolic subgroup* is any subgroup of the form gA_Tg^{-1} for some $T \subseteq V(\Gamma)$ and $g \in A_\Gamma$.

There are many long-standing open questions about Artin groups. For example, it is unknown whether Artin groups are torsion-free, have solvable word problem, or satisfy the famous $K(\pi, 1)$-conjecture. In many cases for which these questions have been answered, we also have an understanding of the structure of their parabolic subgroups.

In geometric group theory, we often study actions of groups on hyperbolic, or more generally, nonpositively curved spaces. CAT(0) spaces have proven to be particularly useful, but this property can be hard to check in dimensions bigger than 2. For this work, we use a combinatorial version of nonpositive curvature known as systolicity, developed by Januszkiewicz and Świątkowski [3] and independently by Haglund [2]. CAT(0)-ness implies unique geodesics, while systolicity does not. But both properties imply contractibility and the following key property: If G acts on X without inversions, and a subgroup $H \leq G$ fixes two vertices in X, then H pointwise fixes the (resp. every) geodesic between the vertices.

An Artin group A_Γ is called *locally reducible* if Γ does not contain triangles of the form 2-3-3, 2-3-4, or 2-3-5. In other words, the only finite type subgraphs of order 3 are of the form 2-2-k for $k \geq 2$. In the case of locally reducible Artin groups, Charney proved that the Deligne complex with the Moussong metric is CAT(0), which implies the $K(\pi, 1)$-conjecture [1].

We will define the 2-complete Artin complex, show that it is systolic for locally reducible Artin groups, and use this to show that many locally reducible Artin groups are acylindrically hyperbolic. We start with a modification of Γ. Let $\widehat{\Gamma}$ be the graph obtained from Γ by deleting all edges not labeled by 2. We are left with the same set of vertices and a subset of the original edges. The *2-complete Artin complex* of A_Γ, denoted $\widehat{X_\Gamma}$, is the simplicial complex with vertices corresponding to left cosets of standard parabolic subgroups of the form $A_{\Gamma \setminus T}$ where T is the set of vertices in a connected component of $\widehat{\Gamma}$. A collection of vertices spans a simplex if the associated cosets have collective nonempty intersection.

Theorem. *Let A_Γ be a locally reducible Artin group. If there are at least three connected components in $\widehat{\Gamma}$, then $\widehat{X_\Gamma}$ is systolic.*

We use this property to show that most parabolic subgroups of a locally reducible Artin group are weakly malnormal. A subgroup $H \leq G$ is said to be *weakly malnormal* if $\exists g \in G$ such that $|H \cap gHg^{-1}| < \infty$. We say a parabolic subgroup is *2-complete* if it can be written as $gA_U g^{-1}$ where $g \in A_\Gamma$ and U is a union of connected components of $\widehat{\Gamma}$. Given a parabolic subgroup P, a *2-completion* of P is any 2-complete parabolic subgroup which contains P.

Theorem. *Let A_Γ be a locally reducible Artin group and suppose $\widehat{\Gamma}$ has at least two connected components. Then any parabolic subgroup P of A_Γ which has a 2-completion that is not all of A_Γ is weakly malnormal.*

We will give the idea of the proof for a 2-complete, standard parabolic subgroup of the form $A_{\Gamma \setminus T}$ where $T = \{t_1, \ldots, t_n\}$ is the set of generators from some connected component of $\widehat{\Gamma}$. Let $t = t_1 \ldots t_n$. Then in the 2-complete Artin complex, there are paths of length 2 from the vertex corresponding to $A_{\Gamma \setminus T}$ to the vertex corresponding to $tA_{\Gamma \setminus T}$ (and each of these paths is a combinatorial geodesic). Since $\widehat{X_\Gamma}$ is systolic, these paths are fixed pointwise. We use this to show that the intersection of the stabilizers of the vertices is trivial, hence $A_{\Gamma \setminus T}$ is weakly malnormal. Using the fact that these parabolic subgroups are weakly malnormal, we can apply theorems of Martin [4] and Minasyan and Osin [5] to reach the following result.

Theorem. *Let A_Γ be a locally reducible Artin group such that $\widehat{\Gamma}$ has at least two connected components.*

(1) *If A_Γ has a maximal finite-type subgroup which is dihedral and which has a 2-completion that is not all of A_Γ, then A_Γ is acylindrically hyperbolic.*
(2) *If A_Γ splits as an amalgamated product $A_{\Gamma_1} *_{A_{\Gamma_1 \cap \Gamma_2}} A_{\Gamma_2}$ such that there is a 2-completion of $A_{\Gamma_1 \cap \Gamma_2}$ that is not all of A_Γ, then A_Γ is acylindrically hyperbolic.*

References

[1] R. Charney, *The Tits conjecture for locally reducible Artin groups*, Int. J. Algebra Comput (2000).
[2] F. Haglund, *Complexes simpliciaux hyperboliques de grande dimension*, Prepublication Orsay (2003).
[3] T. Januszkiewicz and J. Świątkowski, *Simplicial nonpositive curvature*, Publications Mathématiques de l'IHÉS (2006).
[4] A. Martin, *On the acylindrical hyperbolicity of the tame automorphism group of SL2(C)*, Bulletin of the London Mathematical Society (2017).
[5] A. Minasyan and D. Osin, *Acylindrical hyperbolicity of groups acting on trees*, Mathematische Annalen (2015).
[6] H. Van der Lek, *The homotopy type of complex hyperplane complements*, *thesis* (1983).

Calculating braid group actions on categories

Asilata Bapat
(joint work with Anand Deopurkar)

We demonstrate some SageMath code to calculate the action of Artin braid groups on 2-Calabi–Yau triangulated categories arising from undirected graphs.

Let Γ be an undirected graph. Let B_Γ be the (simply-laced) Artin group associated to Γ. There is a 2-Calabi–Yau triangulated category $\mathcal{C}_\Gamma$ associated to Γ, constructed from the zig-zag algebra of (the doubled quiver of) Γ. The precise definition of $\mathcal{C}_\Gamma$ may be found in, e.g. [2, 3] (for Dynkin type A) and [1] (all types). There is a weak action of the Artin group B_Γ on the category $\mathcal{C}_\Gamma$. In this action, the generator σ_i of B_Γ acts by the spherical twist functor in the object P_i (see e.g. [2]).

In this talk, we demonstrate the example where Γ is the Dynkin graph of type A_4. However, the code is equipped for similar calculations for any other simple undirected graph. The code sets up the zig-zag algebra for the given input graph Γ. It creates the basic projective objects P_i, as well as the basic spherical twist operations and their inverses.

We demonstrate how to compute morphisms between objects and apply spherical twists and inverse spherical twists to objects, starting with the generating objects P_i. In particular, we check that the relations of the braid group are satisfied by this action.

Further, we can input the data of a generic standard Bridgeland stability condition (technically, the salient part of the data of a slicing of a Bridgeland stability condition). This is input as a list of the stable objects of the standard heart, from lowest to highest phase. Based on this input, we demonstrate how to compute the Harder–Narasimhan filtration of a given object. We can use this calculation, e.g., to compute examples of the mass growth of objects after applying successive different spherical twists.

We discuss some further possible generalisations as future questions, including some discussion of non-simply-laced type, and how to input the data of a stability condition by listing just the simple objects of the heart rather than all the stable objects.

References

[1] A. Bapat, A. Deopurkar, and A. M. Licata, *A Thurston compactification of the space of stability conditions*, (2020). arXiv:2011.07908.

[2] P. Seidel and R. Thomas, *Braid group actions on derived categories of coherent sheaves.*, Duke Math. J., 108 (2001), pp. 37–108.

[3] R. P. Thomas, *Stability conditions and the braid group*, Commun. Anal. Geom., 14 (2006), pp. 135–161.

Artin groups in symplectic topology

Ailsa Keating

The purpose of this talk was to give an introduction to Artin group actions in symplectic topology, aimed at non-experts. These actions were first studied for braid groups in major works of Seidel and collaborators (see [6, 10]), building on an observation of Arnol'd.

Group actions on plumbings. Given a graph G, possibly with multiple edges, we can associate to it:

(i) An Artin group A_G, with a generator s_i for each vertex i of G, and constants μ_{ij} equal to 2 if there are no edges between i and j; 3 if there is a single edge between them; and ∞ if there are two or more of them.

(ii) In each dimension n, an open symplectic manifold X_G^n given by taking a copy of T^*S^n for each vertex i of G, say T^*S_i, and plumbing them according to the edges of G (for $n = 1$ some auxiliary decorations are required).

There is a natural map from A_G to $\pi_0 \mathrm{Symp}_c X_G^n$, the symplectic mapping class group of X_G^n, given by mapping s_i to the Dehn twist in the Lagrangian S_i.

Fukaya categories. The action by symplectomorphisms induces a representation

$$A_G \to \mathrm{Auteq}\,\mathrm{Fuk}\,X_G^n.$$

Here $\mathrm{Fuk} X_G^n$ can denote several possible 'flavours' of the Fukaya category of X_G^n. In its most basic form, it is generated by the S_is. For $n \geq 3$, this particular category is known to agree with the Ginzburg CYn category associated to the graph (in algebraic terms: the relevant A_∞ category is formal). In this case, the action of A_G precisely matches the 'categorical' action discussed in the talks of Licata and Deopurkar. One advantage of the geometric viewpoint is that one may instead consider actions of 'larger' Fukaya categories, for instance ones whose objects include non-compact Lagrangians; an entry-point for algebraists is [2].

Faithfulness: known cases. As with the purely categorical framework, the group action is conjectured to be faithful whenever the Fukaya category is formal, but this is only known for a handful of cases: ADE and affine A [6, 1, 8, 4, 3].

Finally, a word of warning: for $n = 1$, faithfulness of the action fails [11, 7]; in known cases, this can be interpreted as a failiure of formality of the A_∞ category [9, 5].

References

[1] C. Brav and H. Thomas *Braid groups and Kleinian singularities*, Math. Ann. **351** (2011), no. 4, 1005–1017.

[2] T. Etgü and Y. Lekili *Koszul duality patterns in Floer theory*, Geom. Topol. **21** (2017), 3313–3389.

[3] A. Gadbled, A.-L. Thiel and E. Wagner *Categorical action of the extended braid group of affine type A*, Commun. Contemp. Math.**19** (2017), no.3, 1650024.

[4] A. Keating *Dehn twists and free subgroups of symplectic mapping class groups*, J. Topol. **7** (2014), no. 2, 436–474.

[5] A. Keating, *On symplectic stabilisations and mapping classes*, Bull. Lond. Math. Soc. **54** (2002), 718–736.
[6] M. Khovanov, P. Seidel, *Quivers, Floer cohomology and braid group actions*, J. Amer. Math. Soc. **15** (2002), 203–271.
[7] C. Labruère, *Generalized braid groups and mapping class groups*, J. Knot Theory Ramifications **6** (1997), no. 5, 715–726.
[8] Y. Qiu and J. Woolf *Contractible stability spaces and faithful braid group actions*, Geom. Topol. **22** (2018), no.6, 3701–3760.
[9] P. Seidel *Fukaya categories and Picard-Lefschetz theory*, Zur. Lect. Adv. Math., European Mathematical Society (EMS), Zürich, 2008, viii+326 pp.
[10] P. Seidel, R. Thomas, *Braid group actions on derived categories of coherent sheaves*, Duke Math. J. **108** (2001), 37-108.
[11] B. Wajnryb, *Artin groups and geometric monodromy*, Invent. Math. **138** (1999), no. 3, 563–571.

The spine of untwisted Outer space for RAAGs

Gabriel Corrigan

In this talk, we are only concerned with *right-angled Artin groups* (RAAGs): Artin groups where the constants m_{ij} may only be either 2 or ∞. Our convention is that a RAAG is communicated by a simplicial, unlabelled graph, where two vertices are adjacent if and only if their corresponding generators commute. Hence the graph with n vertices and no edges encodes the free group F_n, while the complete graph on n vertices corresponds to the free abelian group $\mathbb{Z}^n$; in this way, RAAGs can be seen as a natural interpolation between free and free abelian groups.

RAAGs form a special subclass of Artin groups. Their study often involves techniques that do not work more generally - for example, the use of CAT(0) geometry can be particularly fruitful. Consequently, RAAGs are less mysterious than general Artin groups - for example, the $K(\pi, 1)$ conjecture is known to hold, and RAAGs are known to be biautomatic (that is, the word problem is solvable by finite state automata). Despite these nice properties, and the fact that RAAGs are particularly easy to describe, they can still admit rich structure. For example, the fundamental group of a closed hyperbolic 3-manifold always virtually embeds in a RAAG - this was pivotal to Agol's proof of the *virtual Haken conjecture* [1].

In 1986, Culler and Vogtmann [2] constructed what became known as *Culler-Vogtmann Outer space*, CV_n, which allowed for a geometric study of $Out(F_n)$. In a recent series of papers, Charney and Vogtmann, along with various collaborators [3, 4, 5], have constructed an 'Outer space' for RAAGs. That is, given a RAAG A_Γ, they construct an associated contractible complex $\mathcal{O}_\Gamma$ upon which $Out(A_\Gamma)$ acts properly.

Untwisted Outer space. In this talk we will be most concerned with one of the intermediate steps in this construction: the *untwisted Outer space* Σ_Γ, as constructed in [4]. Laurence and Servatius [6, 7] provide a set of generators for $Out(A_\Gamma)$; one family of generators are the so-called *twists*. The subgroup generated by all generators of $Out(A_\Gamma)$ except the twists is called the *untwisted subgroup*, and denoted $\mathcal{U}(A_\Gamma)$. Σ_Γ is a contractible complex upon which $\mathcal{U}(A_\Gamma)$ acts properly. Moreover, it has a natural deformation retract K_Γ called its *spine*; this is a contractible cube complex upon which $\mathcal{U}(A_\Gamma)$ acts both properly and cocompactly. Hence (see [8]), the virtual cohomological dimension of the untwisted subgroup, $\text{VCD}\,(\mathcal{U}(A_\Gamma))$, is bounded above by the dimension of the spine K_Γ.

Free groups admit no twists, so in this case the untwisted Outer space is precisely the original Culler-Vogtmann Outer space. It is easy to verify that $dim(K_{F_n}) = 2n-3$, and also that $Out(F_n)$ has a free abelian subgroup of rank $2n-3$; hence, $\text{VCD}(Out(F_n)) = 2n-3$. However, for general RAAGs, the story is not so easy. Millard and Vogtmann [9] provide a way of finding large-rank free abelian subgroups of $\mathcal{U}(A_\Gamma)$ - thus obtaining lower bounds on $\text{VCD}\,(\mathcal{U}(A_\Gamma))$. In many cases these match $dim(K_\Gamma)$, but not always. They impose a condition on the graph Γ which guarantees that if these bounds do not match, then one may tighten the upper bound by 1. Our main theorem is an extension of this result for certain specific families of graphs Γ. In these cases, one can tighten the upper bound so that it matches the lower bound. In particular, in these examples, the gap between $dim(K_\Gamma)$ and $\text{VCD}\,(\mathcal{U}(A_\Gamma))$ can be arbitrarily large.

Our strategy (an extension of that in [9]) is to find free faces in the cube complex K_Γ that we may (equivariantly) retract, to obtain a complex of smaller dimension which still has a proper and cocompact action of $\mathcal{U}(A_\Gamma)$. The challenge is to find enough free faces so as to admit a retraction which sufficiently reduces the dimension of this complex. This inspires the following question. Although the spine K_Γ had a very natural description, it was 'larger than expected'; on the other hand, the complex obtained from this retraction process is the 'correct dimension' - but is there a more natural intrinsic description of it? One may hope that finding a more natural construction of this complex could lead to generalisations of this technique, and better understanding of the virtual cohomological dimension, at least in the untwisted case. We make one final remark that to date, there is no analogous 'spine' for the full Outer space $\mathcal{O}_\Gamma$.

References

[1] I. Agol *The virtual Haken conjecture*, Docu. Math. **18** (2013), 1045–1087.

[2] M. Culler and K. Vogtmann *Moduli of graphs and automorphisms of free groups*, Invent. Math. **84** (1986), 91–119.

[3] J. Crisp, R. Charney, and K. Vogtmann *Automorphisms of two-dimensional right-angled Artin groups*, Geom. Topol. *11* (2007), 2227–2264.

[4] R. Charney, N. Stambaugh, and K. Vogtmann *Outer space for untwisted automorphisms of right-angled Artin groups*, Geom. Topol. **21** (2017), no. 2, 1131–1178.

[5] C. Bregman, R. Charney, and K. Vogtmann *Outer space for RAAGs*, Duke Math. J. **172** (2023), no. 6, 1033–1108.

[6] H. Servatius *Automorphisms of graph groups*, J. Alg. **126** (1989), no. 1, 34–60.

[7] M. Laurence *A generating set for the automorphism group of a graph group*, J. Lond. Math. Soc. **52** (1995), no. 2, 318–334.

[8] K. Brown *Cohomology of Groups*, Graduate Texts in Mathematics **87** (1982), Springer-Verlag New York, x+309 pp.

[9] B. Millard and K. Vogtmann *Cube complexes and abelian subgroups of automorphism groups of RAAGs*, Math. Proc. Camb. Phil. Soc. **170** (2019), no. 3, 523–547.

Stability manifolds and finite group actions

Hannah Dell

(joint work with Edmund Heng, Antony Licata)

Given a smooth projective complex variety X, one can associate an invariant, $\mathrm{Stab}(X)$, called the stability manifold. In this talk, we will see the role that this invariant plays in algebraic geometry, and in particular how it can be used to detect geometric properties when X admits a free action by a finite group.

1. Motivation: Geometric stability conditions

In this workshop so far we've seen stability conditions arising from singularities and Artin groups. Now let's see an example on a smooth complex projective curve.

Example 1. Let $\mathcal{D} = D^b\mathrm{Coh}(C)$. Then $\sigma = (\mathrm{Coh}(C), -\deg(E) + i\mathrm{rank}(E))$ is a stability condition. Let $0 \neq E \in \mathrm{Coh}(C)$. Then we can define the slope with respect to σ as $\mu_\sigma(E) = +\infty$ if $\mathrm{rank}(E) = 0$, and $\mu_\sigma(E) = \deg(E)/\mathrm{rank}(E)$ otherwise. Then E is σ-stable if and only if any non-zero non-trivial subsheaf F satisfies $\mu_\sigma(F) < \mu_\sigma(E)$. The latter matches the original definition of slope-stability for vector bundles, which was used to classify them [4]. Examples of σ-stable sheaves are line bundles, $\mathcal{O}(n)$, and all skyscraper sheaves of points, $\mathcal{O}_x$.

From now on, we assume X is a smooth projective complex variety. Geometric stability conditions know about the points/ geometry of your variety:

Definition 2. Write $\mathrm{Stab}(X) = \mathrm{Stab}(D^b\mathrm{Coh}(X))$. Then $\sigma \in \mathrm{Stab}(X)$ is *geometric* if for all points $x \in X$, $\mathcal{O}_x$ is σ-stable.

Question 3. *Do there exist non-geometric stability conditions?*

The short answer is "sometimes". We summarise what is known in the literature in the table below, see [1, §1.4] for further details and references.

$\dim X$	$\mathrm{Stab}^{\mathrm{geo}}(X)$	$\mathrm{Stab}(X) \neq \mathrm{Stab}^{\mathrm{geo}}(X)$?
1	$\cong \mathbb{C} \times \mathbb{H}$	$\mathrm{Stab}(\mathbb{P}^1) \cong \mathrm{Stab}(\mathrm{rep}(\bullet = \bullet)) \cong \mathbb{C}^2$
2	controlled by invariants of sheaves on X	$\mathbb{P}^2$, K3 surfaces, rational surfaces, $X \supset C$ rational curve s.t. $C^2 < 0$
≥ 3	$\neq \emptyset$ for some 3folds and $\mathbb{P}^n$	$\mathbb{P}^n$

This leads us to ask what the pattern is, i.e. which geometric properties lead to geometric and non-geometric stability conditions? The first general answer was given by [3, Theorem 1.1]. They showed that if X has finite Albanese morphism alb_X (i.e. a finite map to an abelian variety), then $\mathrm{Stab}(X) = \mathrm{Stab}^{\mathrm{geo}}(X)$.

Question 4 ([3, Q. 4.11])**.** *If* alb_X *is not finite, then is* $\mathrm{Stab}(X) \neq \mathrm{Stab}^{\mathrm{geo}}(X)$*?*

In all known examples, the answer to Question 4 was positive. Our goal is to study a different flavour of examples: *free quotients*, i.e. quotients $Y = X/G$ of varieties by the free action of a finite group. Suppose alb_X is finite while alb_Y is not - this occurs in several examples including Beauville-type and bielliptic surfaces. To answer Question 4 for Y, we need a way to compare $\mathrm{Stab}(Y)$ with $\mathrm{Stab}(X)$.

2. Actions on categories

Let $\mathcal{D}$ be an additive, $\mathbb{C}$-linear category. To a finite group G we can associate a monoidal category $\mathrm{Cat}(G)$, where the objects are group elements, the morphisms are identities, and the tensor product is given by group multiplication.

Definition 5. An *action* of G on $\mathcal{D}$ is an additive monoidal functor $\phi\colon \mathrm{Cat}(G) \to \mathrm{End}(\mathcal{D})$, and we write $\phi_g = \phi(g)$ for $g \in G$. This allows us to define a new category, $\mathcal{D}_G$, the *G-equivariantization*, whose objects are pairs (E, λ) where $E \in \mathcal{D}$ is G-invariant and $\lambda = \{\lambda_g\}_{g\in G}$ is a choice of isomorphism for each g, $\lambda_g\colon E \to \phi_g(E)$.

This is equivalent to the definition Tony Licata gave of a strict action in his mini-course. When G is abelian, there is an action of $\widehat{G} = \mathrm{Hom}(G, \mathbb{C}^\times)$ on $\mathcal{D}_G$. Now assume $\mathcal{D}$ is triangulated and G acts on $\mathcal{D}$ such that each ϕ_g is exact. G also acts on the stabiliy manifold via $\phi_g \cdot (\mathcal{A}, Z) = (\phi_g(\mathcal{A}), Z \circ \phi_g)$. Write $\mathrm{Stab}(\mathcal{D})^G$ for the stability conditions that are fixed by this action. There is a homeomorphism $\mathrm{Stab}(\mathcal{D})^G \cong \mathrm{Stab}(\mathcal{D}_G)^{\widehat{G}}$, see e.g. [1, Lemma 2.23]. Given $\sigma \in \mathrm{Stab}(\mathcal{D})^G$, we obtain $\sigma' \in \mathrm{Stab}(\mathcal{D}_G)^{\widehat{G}}$, where (E, λ) is σ'-semistable if and only if E is σ-semistable.

Theorem 6 ([1, Theorem 3.9, Corollary 3.10])**.** *Suppose $Y = X/G$ is a free quotient with G abelian and* alb_X *finite. Then* $\mathrm{Stab}^\dagger(Y) := \mathrm{Stab}(\mathcal{D}_G)^{\widehat{G}} \subseteq \mathrm{Stab}^{\mathrm{geo}}(Y)$, *with equality if* $\dim(Y) = 2$*. Moreover,* $\mathrm{Stab}^\dagger(Y)$ *is open and closed.*

This tells us that if Y is a Beauville-type or bielliptic surface, then either Question 4 is false, or $\mathrm{Stab}(Y)$ is disconnected. Both of these cases would be surprising!

Together with Edmund and Tony, we are extending this to non-abelian groups. The main task is to understand what to replace the $\widehat{G}$-action with. In this case, the simple representations are no-longer one-dimensional. We can still tensor G-equivariant objects with representations, but $\mathrm{rep}(G)$ is not a group. It is nonetheless a "nice" monoidal category called a *fusion* category, see [2, Definition 2.1].

Definition 7. An *action* of a fusion category $\mathcal{C}$ on $\mathcal{D}$ is an additive monoidal functor $\phi\colon \mathcal{C} \to \mathrm{End}(\mathcal{D})$.

In [2, Theorem 4.8], we show that $\mathrm{Stab}(\mathcal{D})^G \cong \mathrm{Stab}_{\mathrm{rep}(G)}(\mathcal{D}_G)^{\widehat{G}}$, the submanifold of stability conditions that behave well under the $\mathrm{rep}(G)$-action. In the second version of our paper, we will prove that Theorem 6 also generalises, providing further evidence that Question 4 may be false. If this is the case, then which other geometric properties govern non-geometric stability conditions?

References

[1] H. Dell, *Stability conditions on free abelian quotients*, arXiv: 2307.00815.

[2] H. Dell, E. Heng, A. Licata *Fusion-equivariant stability conditions and Morita duality*, arXiv:2311.06857.

[3] L. Fu, C. Li, X. Zhao, *Stability manifolds of varieties with finite Albanese morphisms*, Transactions of the American Mathematical Society **375** (2022), 5669–5690.

[4] D. Mumford, *Projective invariants of projective structures and applications*, Proc. Internat. Congr. Mathematicians (Stockholm, 1962), 526–530.

Homology of configuration spaces of surfaces modulo an odd prime

Andrea Bianchi

The talk is based on joint work with Andreas Stavrou [2]. Let $g \geq 0$, and let S be a compact, connected, oriented surface of genus g with one boundary curve. We let $C_n(S)$ be the *unordered* configuration spaces of n points in S: it is the quotient of $F_n(S) = \{(p_1, \dots, p_n) \in S^{\times n} \mid p_i \neq p_j \ \forall i \neq j\}$ by the natural, free action of the symmetric group $\mathfrak{S}_n$. Configuration spaces play a prominent role in the study of manifolds in general; when $S = D$ is a disc, the space $C_n(D)$ is a classifying space for the Artin group of type A_{n-1} (the standard braid group on n strands); when S is an aspherical surface (e.g. in our assumptions on genus g and one boundary curve), then the space $C_n(S)$ is aspherical, and its fundamental group deserves the name of *braid group of S on n strands.*

We are interested in computing the homology of $C_n(S)$ with coefficients in a field. It is convenient to consider the totality of configuration spaces, i.e. the disjoint union $C_\bullet(S) = \coprod_{n\geq 0} C_n(S)$: then $C_\bullet(D)$ has a natural structure of E_2-algebra, and $C_\bullet(S)$ has a natural structure of E_1-module over $C_\bullet(D)$; passing to homology with coefficients in any ring, $H_*(C_\bullet(D))$ is naturally a graded commutative ring, and $H_*(C_\bullet(S))$ is naturally a module over $H_*(C_\bullet(D))$.

The homology groups $H_*(C_\bullet(D))$ have been computed with different coefficients by Arnol'd [1] ($\mathbb{Q}$), Fuchs [5] ($\mathbb{F}_2$) and, independently, Cohen and Weinstein [7, 4] ($\mathbb{F}_p$ with p odd). The ring structure is the following in the three cases:

- $H_*(C_\bullet(D); \mathbb{Q}) \cong \mathbb{Q}[\epsilon, \alpha]$, with $\epsilon \in H_0(C_1(D); \mathbb{Q})$, $\alpha \in H_1(C_2(D); \mathbb{Q})$;
- $H_*(C_\bullet(D); \mathbb{F}_2) \cong \mathbb{F}_2[\epsilon, \gamma_1, \gamma_2, \dots]$, with $\epsilon \in H_0(C_1(D); \mathbb{F}_2)$ and with $\gamma_i \in H_{2^i-1}(C_{2^i}(D); \mathbb{F}_2)$ for $i \geq 1$;
- $H_*(C_\bullet(D); \mathbb{F}_p) \cong \mathbb{F}_p[\epsilon, \alpha_0, \beta_0, \alpha_1, \beta_2, \dots]$, with $\epsilon \in H_0(C_1(D); \mathbb{F}_p)$ and with $\alpha_i \in H_{2p^i-1}(C_{2p^i}(D); \mathbb{F}_p)$ and $\beta_i \in H_{2p^{i+1}-2}(C_{2p^{i+1}}(D); \mathbb{F}_p)$ for $i \geq 0$.

For $g \geq 1$, we want now to compute $H_*(C_\bullet(S); \mathbb{F}_p)$ as a module over the ring $H_*(C_\bullet(D); \mathbb{F}_p) = \mathbb{F}_p[\epsilon, \alpha_0, \beta_0, \alpha_1, \beta_2, \dots]$. For $i \geq 0$ we introduce the following cyclic modules over the given ring:

- $S_i := \mathbb{F}_p[\epsilon, \alpha_0, \beta_0, \alpha_1, \beta_2, \dots]/(\alpha_0, \dots, \beta_{i-1})$;
- $T_i := \mathbb{F}_p[\epsilon, \alpha_0, \beta_0, \alpha_1, \beta_2, \dots]/(\alpha_0, \dots, \alpha_i)$.

For $k \geq 0$ and for any field $\mathbb{F}$ we denote by $\mathcal{H}(k)$ the vector space $H_1(S; \mathbb{F}) \cong \mathbb{F}^{2g}$, put artificially in homological degree k, and we denote by $\mathrm{Sym}(\mathcal{H}(k))$ the free graded commutative algebra generated by $\mathcal{H}(k)$; we consider $\mathrm{Sym}(\mathcal{H}(k))$ as a plain graded vector space.

The main result stated in the talk is that $H_*(C_\bullet(S); \mathbb{F}_p)$ is isomorphic to a finite direct sum of suitable shifts of modules of the form $S_i \otimes \mathrm{Sym}(\mathcal{H}(2))$ and $T_i \otimes \mathrm{Sym}(\mathcal{H}(2))$; the direct sum can be made explicit, and for instance only values of i such that $p^i \leq g$ can occur.

For comparison, Bödigheimer and Cohen [3] have computed $H_*(C_\bullet(S); \mathbb{Q})$: also in this case one obtains a finite direct sum of suitable shifts of modules of the form $\mathbb{Q}[\epsilon, \alpha] \otimes \mathrm{Sym}(\mathcal{H}(2))$ and $\mathbb{Q}[\epsilon, \alpha]/\alpha \otimes \mathrm{Sym}(\mathcal{H}(2))$.

The computation of $H_*(C_\bullet(S);\mathbb{F}_2)$ is implicit in the work of Löffler and Milgram [6]: $H_*(C_\bullet(S);\mathbb{F}_2)$ is isomorphic to the free module $H_*(C_\bullet(D);\mathbb{F}_2)\otimes \mathrm{Sym}(\mathcal{H}(1))$.

Our computation over $\mathbb{F}_p$ is based on a cell stratification of the configuration spaces $C_n(S)$ which is similar in spirit to the cell stratification of $C_n(D)$ used by Fuchs and Weinstein for their computations; in fact this cell stratification (or rather, the dual cell structure on the homotopy type of $C_n(D)$) coincides with the Salvetti complex of the Coxeter graph of type A_{n-1}.

It would be interesting to find a good definition of configuration spaces depending on a generic manifold M and a generic Coxeter diagram, recovering $C_n(M)$ in the case of A_{n-1}.

References

[1] V. I. Arnol'd, *Certain topological invariants of algebraic functions*, Trudy Mosk. Mat. Obshch. 21 (1970), 27–46.

[2] A. Bianchi and A. Stavrou, *Homology of configuration spaces of surfaces modulo an odd prime*, (2023), arXiv:2307.08664.

[3] C.-F. Bödigheimer and F. R. Cohen, *Rational cohomology of configuration spaces of surfaces*, in "Algebraic Topology and Transformation Groups – Proceedings of a Conference Held in Göttingen, FRG, August 23-29, 1987", Springer (1988), 7–13.

[4] F. R. Cohen, T J. Lada and J. P. May, *The homology of iterated loop spaces*, Springer, Lecture Notes in Mathematics 533 (1976).

[5] D. B. Fuchs, *Cohomologies of the braid group mod 2*, Funktsional. Anal. i Prilozhen. 4:2, (1970), 143–151.

[6] P. Löffler and R. J. Milgram, *The structure of deleted symmetric products*, Contemporary Math. 78 (1988), 415–424.

[7] F. V. Weinstein, *Cohomology of the braid groups*, Funktsional. Anal. i Prilozhen. 12:2 (1978), 135–137.

$K(\pi,1)$-conjecture for Artin groups via combinatorial non-positive curvature

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Given an Artin group with Dynkin diagram Λ and generating set S, the associated *Artin complex* associated with an Artin group is defined as follows. For each $s\in S$, let $A_{\hat{s}}$ be the standard parabolic subgroup generated by $S\backslash\{s\}$. The *Artin complex*, denoted by Δ_Γ (or Δ_S), is the simplicial complex whose vertex set corresponds to left cosets of $\{A_{\hat{s}}\}_{s\in S}$. A collection of vertices span a simplex if the associated cosets have nonempty common intersection. A vertex of Δ_S has *type* $\hat{s}=S\setminus\{s\}$, if it corresponds to a coset of form $gA_{S\setminus\{s\}}$.

By a result of Godelle and Paris [3], proving the $K(\pi,1)$-conjecture for all Artin groups reduces to proving all Artin complexes are contractible, whenever the associated Artin groups are not spherical.

We explore ways to prove the contractibility of some Artin complexes, based on methods from combinatorial non-positive curvature. The general method has two steps. First we find certain combinatorial conditions on how the simplices are glued together locally, and try to show these conditions imply the contractibility of space. This relies on ideas from earlier work of Chepoi, McCammond, Haettel, Hirai, Lang and Osajda [1, 2, 4, 6]. Second we verify these local conditions are indeed true. The work is still in progress, though the first part is already on Arxiv [5]. We manage to find a very simple local condition which holds true for certain Artin complexes and gives contractibility of the spaces. This proves a large class of Artin groups satisfying $K(\pi, 1)$-conjecture which are not previously known.

Let Λ be a Dynkin diagram which is a tree, with its vertex set S. Let X be the 1-skeleton of Δ_S with its vertex labeling as explained above. We say Δ_S satisfies the *labeled 4-wheel condition* if for any induced 4-cycle in X with consecutive vertices being $\{x_i\}_{i=1}^4$ and their labels being $\{\hat{s}_i\}_{i=1}^4$, there exists a vertex $x \in X$ adjacent to each of x_i such that label $\hat{s}$ of x satisfies that s is in the smallest subtree of Λ' containing all of $\{s_i\}_{i=1}^4$.

We showed that whenever Λ is irreducible spherical, then Δ_S satisfies the labeled 4-wheel condition. And use this to show the following class of Artin groups satisfy the $K(\pi, 1)$-conjecture.

Suppose Λ is a tree Dynkin diagram. Suppose there exists a collection E of open edges with label ≥ 6 such that for each component Λ' of $\Lambda \setminus E$ is spherical. Then A_Λ satisfies the $K(\pi, 1)$ conjecture. Besides this, we also showed the $K(\pi, 1)$-conjecture for new examples of Artin groups associated with reflection groups acting on hyperbolic spaces up to dimension 7.

It is a ongoing work to use this method to understand the $K(\pi, 1)$-conjecture for more general hyperbolic type Artin groups.

References

[1] Thomas Brady and Jonathan P. McCammond. Braids, posets and orthoschemes. *Algebr. Geom. Topol.*, 10(4):2277–2314, 2010.

[2] Jérémie Chalopin, Victor Chepoi, Hiroshi Hirai, and Damian Osajda. Weakly modular graphs and nonpositive curvature. *arXiv preprint arXiv:1409.3892. To appear in Mem. Amer. Math. Soc.*, 2014.

[3] Eddy Godelle and Luis Paris. K(π, 1) and word problems for infinite type artin–tits groups, and applications to virtual braid groups. *Mathematische Zeitschrift*, 272(3):1339–1364, 2012.

[4] Thomas Haettel. A link condition for simplicial complexes, and CUB spaces. *arXiv preprint arXiv:2211.07857*, 2022.

[5] Jingyin Huang. Labeled four cycles and the K(π, 1)-conjecture for Artin groups. *arXiv preprint arXiv:2305.16847*, 2023.

[6] Urs Lang. Injective hulls of certain discrete metric spaces and groups. *Journal of Topology and Analysis*, 5(03), 297–331, 2013.

The galaxy of Coxeter groups

Petra Schwer
(joint work with Yuri Santos Rego)

This report is based on [1], which is dedicated to the memory of Jacques Tits for his influential work on Coxeter groups. It was him who, as an honorary Bourbaki, coined the terms Coxeter group and Coxeter diagram.

1. What is the Coxeter galaxy?

While the isomorphism problem is undecidable in the full universe of groups, the region containing Coxeter groups is fairly well understood and it is expected that the problem is solvable within this class of groups. The *Coxeter galaxy* provides a framework to study this and related, more refined questions. The galaxy itself is is the flag simplicial complex $\mathcal{G}$ whose vertices are (graph-isomorphism classes of) Coxeter systems of finite rank, two of which are connected if their underlying Coxeter groups are isomorphic as abstract groups. Solving the isomorphism problem amounts to algorithmically being able to determine the connected components of the galaxy. By construction $\mathcal{G}$ is organized in layers according to the rank of the Coxeter systems.

Figure 1 provides first examples of Coxeter graphs defining the same underlying (Coxeter) group. These graphs span three simplex in the galaxy with one vertex of rank three, two of rank four and one vertex of rank five.

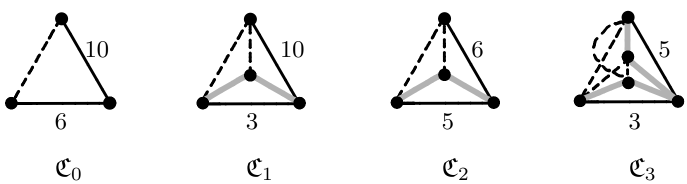

Figure 1. Four complete Coxeter graphs describing a same group. The graphs $\mathfrak{C}_1, \mathfrak{C}_2$ and $\mathfrak{C}_3$ may be obtained from $\mathfrak{C}_0$ by (repeated) applications of blow-ups.

2. How does the galaxy look like locally and globally?

As illustrated by the example, connected components of the galaxy typically span over several layers. The subcomplex spanned by Coxeter systems of rank at most k is denoted by $\mathcal{G}_{\leq k}$. A solution to the isomorphism problem for (finitely generated) Coxeter groups is then equivalent to a solution to all 'height-k restricted isomorphism problems', i.e., for all $\mathcal{G}_{\leq k}$ for every $k \in \mathbb{N}$. We summarize our main structural findings in the following theorem.

Theorem 2.1 (Structure of the Coxeter galaxy $\mathcal{G}$)**.**

(1) *The galaxy $\mathcal{G}$ is a locally finite, infinite dimensional simplicial complex with finite connected components, all of which are simplices.*
(2) *Solving the isomorphism problem reduces to algorithmically computing (a spine of) a proper subcomplex, called the vertical core.*
(3) *The subcomplex $\mathcal{G}_{\leq 3}$ is a 1-dimensional complex and equal to its vertical core. Furthermore, the isomorphism problem is decidable for groups in this subcomplex.*

Many questions remain open: how does the dimension of the layers grow if rank grows? Can one tell from a graph how many vertices are contained in the in intersection of a component with a given layer?

3. How to navigate the galaxy?

Classical approaches to the isomorphism problem involve a) reducing to a specific subclass of Coxeter groups and to b) finding explicit moves, i.e. explicit manipulations, between defining graphs. These moves correspond to edges in the Coxeter galaxy. Although many preliminary results exist, several fundamental problems remain open:

Question 3.1 (Reachability and Coloring Problem)**.** Does there exist a finite list of vertical and horizontal moves such that, for any vertex in the Coxeter galaxy, any other vertex in its connected component can be reached by applying a finite sequence of these moves? Can such an algorithm also output which kinds of moves are needed along a path between vertices of the galaxy?

This question (implicitly) appeared multiple times in the literature and was answered positively in some cases. For example, Howlett and Mühlherr introduced moves called blow-ups. Results by Mihalik, Ratcliffe, and Tschantz imply that they suffice to vertically navigate the galaxy. The current state of the art concerning the corresponding horizontal question revolves around Mühlherr's twist conjecture, which is discussed in detail in the final section of [1].

4. How does profinite rigidity come into play?

The proof of item (3) of Theorem 2.1 uses profinite techniques. Building up on results of Bridson-Conder-reid, we deduche a complete picture of the first three layers $\mathcal{G}_{\leq 3}$ of the galaxy by showing that Coxeter groups in $\mathcal{G}_{\leq 3}$ are profinitely rigid within that family.

Theorem 4.1 (Profinite rigidity in rank ≤ 3)**.**
Coxeter groups with diagrams in $\mathcal{G}_{\leq 3}$ are profinitely rigid within this class, that is, two such groups are isomorphic if and only if their profinite completions agree.

References

[1] Y. Santos Rego, P. Schwer *The galaxy of Coxeter groups*, J. of Algebra, (2023). https://doi.org/10.1016/j.jalgebra.2023.12.006

Polynomials and the Dual Braid Complex

Jon McCammond
(joint work with Michael Dougherty)

The braid groups have several well known classifying spaces. One of the earliest and best known is the quotient of the complement of the complexified braid arrangement by the free action of the symmetric group. This can also be viewed as the space of monic centered polynomials of degree d with distinct roots. Monic means the lead coefficient is 1 and centered means that the next coefficient is 0. The dual braid complex is a modern classifying space constructed from the dual Garside structure on the braid group [1, 3, 4]. It is a compact cell complex with a piecewise Euclidean orthoscheme metric on its simplices. Since both sequences of spaces are classifying spaces for the same sequenes of groups, they are known to be homotopy equivalent, but until recently there has been no explicit natual map that realizes this homotopy equivalence.

The dual braid complex is a quotient of the order complex of the poset $\mathrm{NCP}\textsc{art}_d$ of noncrossing partitions, where the quotient is defined by face identifications. Recall that a partition of the vertex set of a convex d-gon is *noncrossing* if the convex hulls of the vertices in each block of the partition are pairwise disjoint. The partial ordering of these partitions by refinement also encode the ways to factor a d-cycle in the symmetric group into $d-1$ transpositions [2].

In recent work with Michael Dougherty, we construct an explicit homotopy equivalent embedding of the dual braid complex into the space $\mathrm{P}\textsc{oly}^{mc}_d(\mathbb{C}_{\mathbf{0}})$ of monic centered polynomials of degree d with distinct roots [5, 6] and the image of this embedding is the collection of points labeled by polynomials whose critical values lie on the unit circle. The key idea is to use the polynomial interpretation, as Daan Krammer suggested to the second author in 2017.

For each $U \subset \mathbb{C}$, let $\mathrm{P}\textsc{oly}^{mc}_d(U)$ be the collection of monic centered polynomials of degree d where all critical values lies in U. And recall that the Lyashko-Looijenga map, or LL-map, sends a monic centered polynomial to its multiset of critical values. Using the fact that the LL-map is a stratified covering map, we establish homeomorphisms between spaces of such polynomials with critical values in U and those with critical values in V, so long as there is a homotopy from U to V where distinct points remain distinct throughout the homotopy. In particular, we prove the following results characterizing polynomials with critical values in an interval, a rectangle, a circle and an annulus.

Theorem 1 (Intervals)**.** The space $\mathrm{P}\textsc{oly}^{mc}_d(\bullet\!\!-\!\!\bullet)$ of monic centered polynomials of degree d with critical values in a closed interval is homeomorphic to the complex of branched planar lines called "metric banyans" and to $|\mathrm{NCP}\textsc{art}_d|$, the order complex of the noncrossing partition lattice with the orthoscheme metric.

Theorem 2 (Rectangles)**.** The space $\mathrm{P}\textsc{oly}^{mc}_d(\blacksquare)$ of polynomials with critical values in a closed rectangle is a (proper) subcomplex of the direct product of $\mathrm{P}\textsc{oly}^{mc}_d(\bullet\!\!-\!\!\bullet) \times \mathrm{P}\textsc{oly}^{mc}_d(\bullet\!\!-\!\!\bullet)$ called the branched rectangle complex or the basketball complex. It's top-dimensional cells are products of two orthoschemes. It can be viewed as a compactification of the space of all polynomials. It is homeomoprhic to a topoloical closed ball and it is metrically a manifold with corners.

The proof of this theorem uses a left-to-right Morse function to determine a chain in the noncrossing partition lattice, and a top-to-bottom Morse function to determine a second chain in a second noncrossing partition lattice. The compatibility condition between the two is related to the "basketballs" introduced by Martin, Salvitt and Singer in 2007 [7]. In addition, homotopies of subsets of $\mathbb{C}$ where points are allowed to merge but not split can also be lifted to provide quotients and deformations of spaces of polynomials. For example, the polynomials with critical values in circles and annuli are obtained via face identifications on the spaces of polynomials with critical values in intervals and rectangles.

Theorem 3 (Circles)**.** The space $\textsc{Poly}_d^{mc}(\bigcirc)$ of monic centered polynomials of degree d with critical values in a circle is homeomorphic to the complex of branched planar circles called "metric cacti". It can also be viewed as a quotient by face identifications of $\textsc{Poly}_d^{mc}(\bullet\!\!-\!\!\bullet)$. Finally, it is the cell complex derived from the dual Garside structure of the braid group with the orthoscheme metric.

Theorem 4 (Annuli)**.** The space $\textsc{Poly}_d^{mc}(\circledcirc)$ of monic centered polynomials with critical values in a closed annulus is homeomorphic to the branched annulus complex, and it is a face identification of $\textsc{Poly}_d^{mc}(\square)$. It can also be viewed as a compactification of the space $\textsc{Poly}_d^{mc}(\mathbb{C}_{\mathbf{0}})$ of monic centered polynomials with distict roots.

Finally, the deformation retraction from $\mathbb{C}_{\mathbf{0}}$ to the unit circle, shows that the dual braid complex is not only contained in the space of monic centered polynomials with distinct roots. It is also a spine for this space.

Theorem 5 (Deformations)**.** The space $\textsc{Poly}_d^{mc}(\mathbb{C}_{\mathbf{0}})$, the classical classifying space for the braid groups, contains the subspace $\textsc{Poly}_d^{mc}(\bigcirc)$, which is the dual braid complex with the orthoscheme metric. Moreover, the former deformation retracts to the latter, showing that they are homotopy equivalent.

References

[1] D. Bessis, *The dual braid monoid*, Annales Scientifiques de l'École Normale Supérieure **36** (2003), 647–683.

[2] P. Biane, *Some properties of crossings and partitions*, Discrete Mathematics **175** (1997), 41–53.

[3] T. Brady, *A partial order on the symmetric group and new $K(\pi,1)$'s for the braid groups*, Advances in Mathematics, **161** (2001), 20–40.

[4] T. Brady and J. McCammond, *Braids, posets and orthoschemes*, Algebraic and Geometric Topology **10** (2010), 2277-2314.

[5] M. Dougherty and J. McCammond, *Geometric combinatorics of polynomials I: The case of a single polynomial*, Journal of Algebra **607** (2022), 106–138.

[6] M. Dougherty and J. McCammond, *Geometric combinatorics of polynomials II: polynomials and cell structures*, in preparation.
[7] J. Martin, D. Savitt and T. Singer, *Harmonic algebraic curves and noncrossing partitions*, Discrete & Computational Geometry **37** (2007), 267–286.

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