

EXERCISE SET #1

ROBOTS ON GRAPHS

JON MCCAMMOND

1. (Easy) Let Γ be the graph that is the boundary of a hexagon with 6 vertices and 6 edges and orient these edges in a counter-clockwise direction. We construct a new directed graph and (directed) simplicial complex X as follows.
 - The vertices of X are indexed by the ways to place 2 distinct (i.e. labeled) robots on the vertices of Γ (with only one robot per vertex). There are 30 such vertices.
 - Draw a directed edge from a vertex v to a vertex v' in X if the robot configuration v can be moved to the robot configuration v' with each robot moving along at most one edge in the counter-clockwise direction.
 - The length of each edge in X is \sqrt{k} where k is the number of robots that are moving.
 - Add a simplex to each complete subgraph and its metric should be the unique euclidean simplex with these edge lengths. The simplices here are all isosceles right triangles.

Prove that

- (a) this space is non-positively curved,
 - (b) its universal cover is CAT(0),
 - (c) and explicitly describe the global metrics on these spaces.
2. (Easy) Repeat Exercise 1 with indistinguishable (i.e. unlabeled) robots.
 3. (Medium) Repeat Exercises 1 and 2, but with 3, 4, 5 or 6 robots on the 6-cycle instead of just 2, and include both the labeled and the unlabeled cases, as well as the universal cover.
 4. (Hard) Repeat Exercise 1 and 2 with k robots on a n -cycle, labeled and unlabeled, as well as the universal cover.