9. You want to carry out an affine encryption but $\gcd(\alpha, 26) > 1$. Show that if

$$ x_1 = x_2 + \frac{26}{d} \quad (*) $$

then $\alpha x_1 + \beta = \alpha x_2 + \beta \pmod{26}$

Multiplying both sides of equation (*) by $\alpha$ we get $\alpha x_1 = \alpha x_2 + \alpha(\frac{26}{d})$. Since $d$ divides $\alpha$, $\alpha(\frac{26}{d}) = (\frac{\alpha}{d})26$ is a multiple of 26. We can rewrite this equation as $\alpha x_1 - \alpha x_2 = 26n$ where $n$ is an integer. By definition of congruence: $\alpha x_1 \equiv \alpha x_2 \pmod{26}$. We can add any constant $\beta$ to both sides of the congruence:

Therefore:

$$ \alpha x_1 + \beta \equiv \alpha x_2 + \beta \pmod{26} $$