2. Suppose your RSA modulus is 55, e = 3. Note that \( \varphi(55) = \varphi(5)\varphi(11) = 4 \times 10 = 40 \).

   a) The encryption modulus \( d \) must have the property that \( d \cdot e \equiv 1 \pmod{40} \). Given that \( e = 3 \), we quickly see that \( d = 27 \) is a solution; since \( 27 \times 3 = 81 \)

   b) Now, assuming gcd\((m,55)\) = 1 for some message \( m \) we want to show that if \( c \) is the cipher text, then \( m \equiv c^{27} \pmod{55} \) is the plain text. Notice, since \( 27 \times 3 \equiv 1 \pmod{40} \) we have that \( d \cdot e = 1 + 2 \times 40 \). Thus, \( c^{27} \equiv (m^3)^{27} \pmod{55} \) and \( (m^3)^{27} \equiv 1 (1+80) \equiv m^*(m^{40}) \equiv m^*(m^{40})^2 \equiv m^*1^2 \equiv m \pmod{55} \). Notice, that \( m^{40} \equiv 1 \pmod{55} \) by Euler's theorem since gcd\((m,55)\) = 1 implies that \( m^{40} \equiv 1 \pmod{55} \).