Chapter 6 number 23:

If \( \gcd(e, 12345) = 1 \) then we can find a number \( d \) such that \( ed \equiv 1 \pmod{12345} \); that is, there exists an integer \( k \) with \( ed = k \cdot 12345 + 1 \). Then we will raise \( m^e \) to the \( d \) power \( (m^e)^d = m^{k \cdot 12345 + 1} = m \cdot m^{k \cdot 12345} \equiv m \pmod{12345} \) since \( m^{k \cdot 12345} = (m^{12345})^k \equiv 1 \pmod{12345} \). Thus, I would be able to find \( m \) in this case since the spy gave the information \( m^{12345} \equiv 1 \pmod{n} \). In practice \( \gcd(e, 12345) = 1 \) will occur quite often as \( e \) is often chosen to be a large prime such as \( 2^{16} + 1 \), to ensure that it will be relatively prime with \( \phi(n) \). \( \gcd(e, 12345) = 1 \) if \( 12345 \) is the order of \( m \pmod{n} \) (this is not necessarily the case in general), as in this case you will have \( 12345 \) divides \( \phi(n) \) and we also know that \( \gcd(\phi(n), e) \) will be 1.

Note that the prime factorization of 12345 is \( 3 \cdot 5 \cdot 823 \)