7.1  

(a) Let \( p=13 \). Compute \( L_2(3) \)

(b) Show that \( L_2(11) = 7 \)

(a) & (b) We want to find \( x \) such that \( 2^x \equiv 7 \pmod{13} \). Note first that 2 is a primitive root mod 13 because \( 2^6 = 64 \equiv -1 \pmod{13} \). Taking successive powers of 2, we get \( 2^4 = 16 \equiv 3 \pmod{13} \)

So we have that \( x=4 \), and \( L_2(3) = 4 \)

Next, we can check that \( 2^7 = 128 \equiv 11 \pmod{13} \). No other power between 1 and 12 works because 2 is a primitive root \( \pmod{13} \).

Thus, we have that \( L_2(11) = 7 \)

For larger moduli we would want to reduce the number of cases we have to compute using, for example, the technique in section 7.2 to first determine whether \( x \) is even or odd.