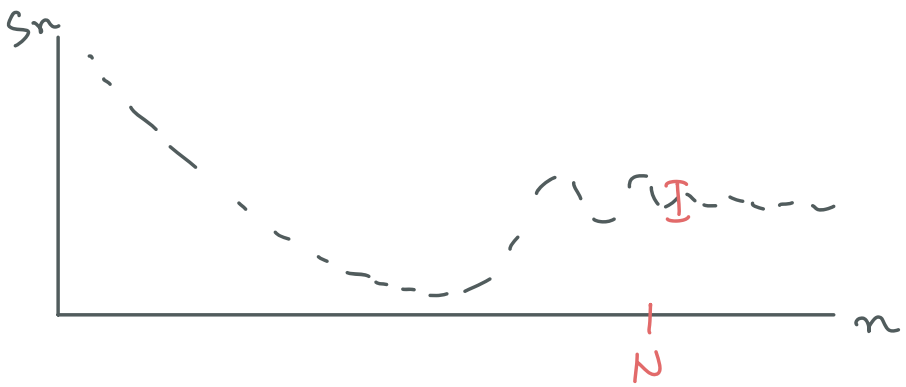
Lecture 10 - Highlights

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Def (Cauchy sequence): A sequence s_n is a **Cauchy sequence** if for all $\varepsilon > 0$, there exists $N \in \mathbb{R}$ s.t. $m, n > N$ ensures $|s_n - s_m| < \varepsilon$



Lemma: Convergent sequences are Cauchy sequences

Lemma: Cauchy sequences are bounded.

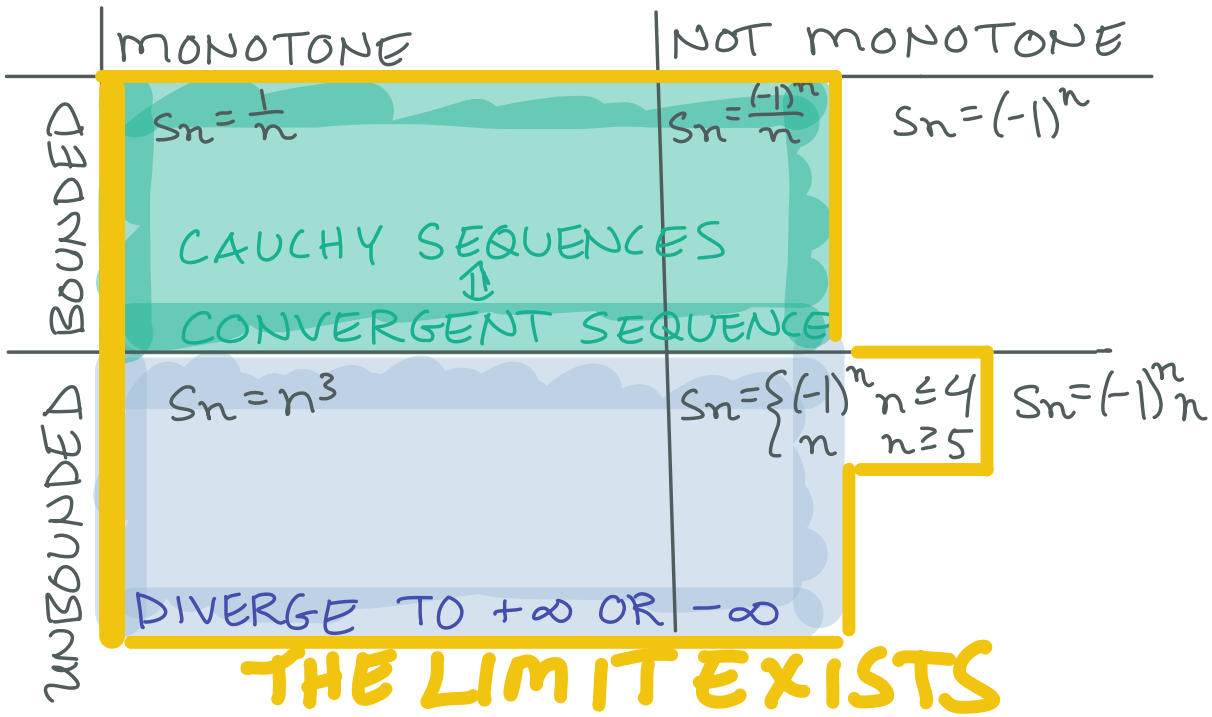
MAJOR THEOREM #4

Thm: A sequence is convergent iff it is Cauchy.

Here you must know what the limit is and show elts of sequence get close to it.

Here you must know that elts of sequence "bunch up". (Don't need to know what they are bunching up around.)

Types of Sequences:



HW Problem:

If a sequence sn satisfies lim sup sn = b, then sn ≤b for all n large enough.

True or False?

$\limsup_{n\to\infty} s_n = b,\ b \in \mathbb{R} \overset{?}{\Longrightarrow} s_n \leq b$ for all n large enough.

Ex:

$s_n = (1, 1, \dots)$, $\lim_{n\to\infty} s_n = \limsup_{n\to\infty} s_n = 1$ True

$s_n = \frac{1}{n}$, $\lim_{n\to\infty} s_n = \limsup_{n\to\infty} = 0 \Rightarrow$ False

=

The claim is false. To see this, consider the sequence $s_n = \frac{1}{n}$. Since the sequence converges to zero, the limit exists, so the limit is the same as the limsup and $\limsup_{n\to\infty} s_n = 0$. However, $\frac{1}{n} > 0 \ \forall n \in \mathbb{N}$, so $\frac{1}{n} \neq 0$ for n large enough.