Reminders:

- Math TED Talk
- HW6 and Practice Midterm 2 posted on website (solutions to Practice Midterm 2 posted Thursday)
- Office hours TODAY 3:15-4:15pm
- No class or office hours on Wednesday (Veterans' Day)
- Midterm 2a on Monday, November 16th
- Midterm 2b on Wednesday, November 18th

Lecture 11

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Ex: $s_n = (1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots)$

$n_k = (2, 4, 6, 8, \dots)$

$s_{n_k} = (\frac{1}{3}, \frac{1}{7}, \frac{1}{9}, \dots)$

Def (subsequence): Consider a sequence s_n. For any sequence n_k of natural numbers satisfying $n_1 < n_2 < n_3 < \dots$, a sequence of the form s_{n_k} is a **subsequence** of s_n.

Remark: We could write s_n as $s(n)$, n_k as $n(k)$, and s_{n_k} as $s(n(k))$.

Lemma: Given a sequence s_n, $n \in \mathbb{N}$, if s_{n_k} is a subsequence, then $n_k \geq k$ for all $k \in \mathbb{N}$.

Def: (subsequential limit) A **subsequential limit** of a sequence s_n is any real number or symbol $+\infty$ or $-\infty$ that is the limit of some subsequence of s_n.

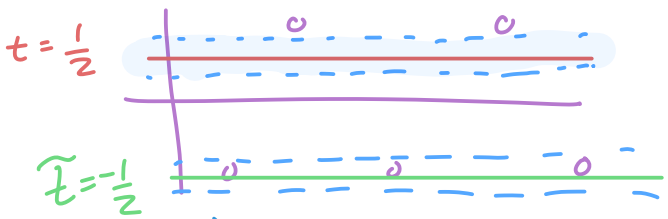


Thm: If a sequence s_n converges to a limit s, then every subsequence also converges to s.

$s_n = (-1)^n$

$t = \frac{1}{2}$

$\tilde{t} = -1$



Thm (main subsequences theorem)

Let s_n be a sequence of real numbers.

(a) Let $t \in \mathbb{R}$

[The set $\{n : |s_n - t| < \varepsilon\}$ is infinite for all $\varepsilon > 0$]

if and only if

[t is a subsequential limit of s_n.]

(b) s_n is unbounded above $\Leftrightarrow$ $+\infty$ is a subseq. limit.

(c) s_n is unbounded below $\Leftrightarrow$ $-\infty$ is a subseq. limit.

Common mistakes on Midterm #1

1) Showing M is an upper bound for a set S doesn't imply $\sup(S) = M$. (Need M is the <u>least</u> upper bound.)

2) "Limit of quotient is quotient of limit"

Suppose t_n, s_n are convergent and $\lim_{n\to\infty} t_n \neq 0$. Then

$$\lim_{n\to\infty} \frac{s_n}{t_n} = \frac{\lim_{n\to\infty} s_n}{\lim_{n\to\infty} t_n}$$

$\Downarrow$

More generally, keep in mind that "the limit exists" $\not\Rightarrow$ "the sequence converges" $\Leftarrow$

Likewise,

$$\lim_{n\to\infty} \frac{an^k + b}{cn^m + d} \neq \frac{\lim_{n\to\infty} an^k + b}{\lim_{n\to\infty} cn^m + d}$$

$$\frac{a\cdot\infty}{c\cdot\infty} = \text{sad teacher} \therefore$$

3) Justify yourself using defn.

"s_n grows exponentially"
"s_n keeps growing and growing"
"s_n is positive and increasing"

Not rigorous argument for proving $\lim_{n\to\infty} s_n = +\infty$.