Lecture 9 - Highlights

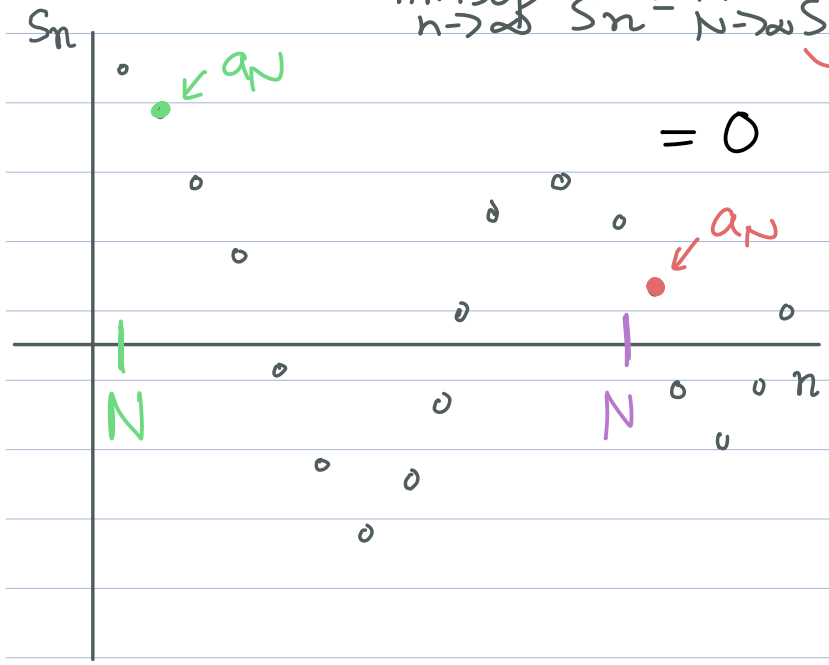
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Thm: Given a sequence s_n,

$$\lim_{n\to\infty} s_n \text{ exists} \iff \liminf_{n\to\infty} s_n = \limsup_{n\to\infty} s_n.$$

Furthermore, if either of these equivalent conditions holds, $\lim_{n\to\infty} s_n = \liminf_{n\to\infty} s_n = \limsup_{n\to\infty} s_n$.

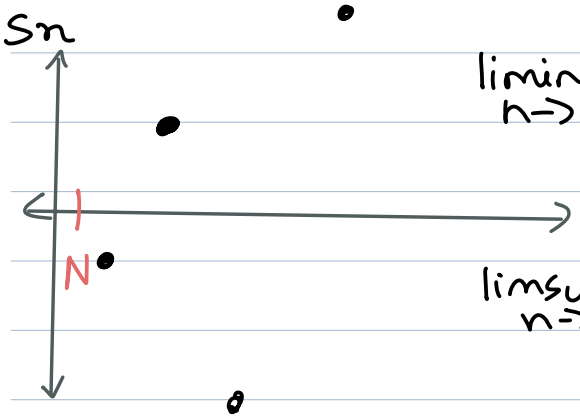
$$\limsup_{n\to\infty} s_n = \lim_{N\to\infty} \underbrace{\sup\{s_n : n > N\}}_{a_N}$$



Example:

Consider $s_n = (-1)^n n$ and $t_n = 2n^3$.

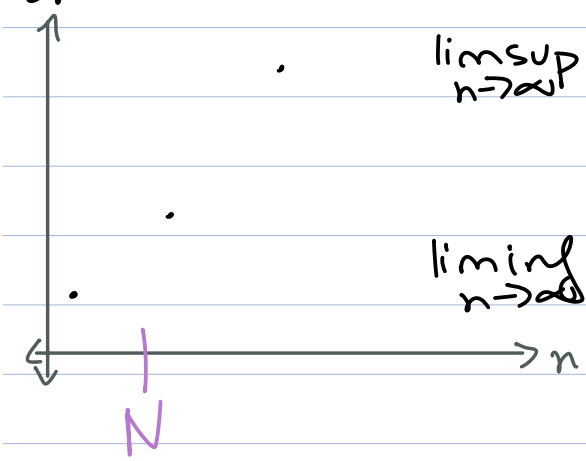
What is $\liminf_{n\to\infty} s_n$? $\liminf_{n\to\infty} t_n$?



$$\liminf_{n\to\infty} s_n = \lim_{N\to\infty} \inf\{s_n : n>N\}$$
$$= \lim_{N\to\infty} \underbrace{(-\infty)}_{b_N}$$
$$= -\infty$$

$$\limsup_{n\to\infty} s_n = \lim_{N\to\infty} \sup\{s_n : n>N\}$$
$$= \lim_{N\to\infty} (+\infty)$$
$$= +\infty$$

$t_n = 2n^3$



$$\limsup_{n\to\infty} t_n = \lim_{N\to\infty} \sup\{t_n : n>N\}$$
$$= \lim_{N\to\infty} (+\infty)$$
$$= +\infty$$

$$\liminf_{n\to\infty} t_n = \lim_{N\to\infty} \inf\{2n^3 : n>N\}$$
$$= \lim_{N\to\infty} \underbrace{2(N+1)^3}_{b_N}$$
$$= +\infty$$