

MATH 117: MIDTERM 1A

Monday, October 26th, 2026

Name: _____

Student ID #: _____

Signature: _____

This is a closed-book and closed-note examination. Calculators are not allowed. Please show your work in the space provided. I will provide scratch paper—other forms of scratch paper are not permitted. If you continue a problem on the back of a page, please write “continued on back”. Partial credit will be given for partial answers. You have 1 hour and 15 minutes.

Question	Points	Score
1	12	
2	14	
Total	26	

Question 1 (12 points)

Let A and B be nonempty bounded subsets of $\mathbb{R}$, and define $A + B = \{a + b : a \in A \text{ and } b \in B\}$.

- (a) State the definition of what it means for a nonempty set $S \subseteq \mathbb{R}$ to be bounded above.
- (b) Prove that, for all $b \in B$, $\sup(A + B) - b$ is an upper bound for A .
- (c) Prove that $\sup(A + B) = \sup A + \sup B$.

Question 2 (14 points)

Prove the following useful lemma:

Lemma. *If r_n and t_n are sequences whose limits exist and $r_n \leq t_n \ \forall n \in \mathbb{N}$, then*

$$\lim_{n \rightarrow +\infty} r_n \leq \lim_{n \rightarrow +\infty} t_n.$$

Do **not** assume that the sequences r_n and t_n converge, only that the limits exist.

