**Def (Cauchy sequence):** A sequence \( \{s_n\} \) is a Cauchy sequence if for all \( \varepsilon > 0 \), there exists \( N \in \mathbb{N} \) such that \( |s_m - s_n| < \varepsilon \) whenever \( m, n > N \).

**Lemma:** Convergent sequences are Cauchy sequences.

**Lemma:** Cauchy sequences are bounded.

**MAJOR THEOREM #4**

**Thm:** A sequence is convergent if and only if it is Cauchy.

Here you must know what the limit is and show the elements of the sequence get close to it.

Here you must know that the elements of the sequence "bunch up." (Don't need to know what they are bunching up around.)
### Types of Sequences

<table>
<thead>
<tr>
<th>BOUNDED</th>
<th>MONOTONE</th>
<th>NOT MONOTONE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$s_n = \frac{1}{n}$</td>
<td>$s_n = \frac{(-1)^n}{n}$</td>
</tr>
<tr>
<td>CAUCHY SEQUENCES</td>
<td>CONVERGENT SEQUENCE</td>
<td></td>
</tr>
</tbody>
</table>
|          | $s_n = n^3$   | $s_n = \left\{ \begin{array}{ll}
(-1)^n & n \leq 4 \\
\frac{1}{n} & n > 25
\end{array} \right.$ | $s_n = (-1)^n$ |
| DIVERGE TO $+\infty$ OR $-\infty$ | THE LIMIT EXISTS |