Name: ____________________________________________
Student ID #: ______________________________________
Signature: __________________________________________

This is a closed-book and closed-note examination. Calculators are not allowed. Please show your work in the space provided. I will provide scratch paper—other forms of scratch paper are not permitted. If you continue a problem on the back of a page, please write “continued on back”. Partial credit will be given for partial answers. You have 1 hour and 15 minutes.

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Let $A$ and $B$ be nonempty bounded subsets of $\mathbb{R}$, and define $A + B = \{a + b : a \in A \text{ and } b \in B\}$.

(a) State the definition of what it means for a nonempty set $S \subseteq \mathbb{R}$ to be bounded above.

(b) Prove that, for all $b \in B$, $\sup(A + B) - b$ is an upper bound for $A$.

(c) Prove that $\sup(A + B) = \sup A + \sup B$. 
Question 2 (28 points)

(a) State the definition of a convergent sequence.

(b) State the definition of what it means for a sequence to diverge to $+\infty$.

(c) Suppose the limit of $t_n$ exists. Prove that

$$\lim_{n \to +\infty} |t_n| = \left| \lim_{n \to +\infty} t_n \right|.$$

(You may use the fact that $|+\infty| = +\infty$ and $|−\infty| = +\infty$.)

(d) Give an example for which $\lim_{n \to +\infty} |t_n|$ exists, but the equality from part (c) does not hold. You do not need to justify your example.