<u>Lecture3</u> - Math movie competition KatyCraig <sup>2024</sup>

Def lordered field A field F is an ordered field it it has an ordering relation so that, for all  $a,b,c \in F:\bigcirc$ 

 $(a)$  either  $a \leq b$  or  $b \leq a$  $(01)$  either  $a \leq b$  or  $b \leq a$  totality<br> $(02)$  if  $a \leq b$  and  $b \leq a$ , then  $a \leq b$  antisymmetry  $(a3)$  if a sb and b sc, then a sc transitivity  $tot$  if  $a 5b$ , then  $at c 5b+c$  addition  $105$  ) if a  $\leq$  b and c  $\geq$  0, then  $ac$   $\leq$  bc multiplication

 $\bigcup_{i\in\mathbb{C}} s_i$  is iner an ordered field F and  $a_i b^{e+1}$  $if$  a  $\leq$  b and a  $\neq$  b, then write a  $\leq$  b

Def (maximum, minimum) Suppose SSF, where<br>Fis an ordered field. . If there exists so S satisfying so ZS for all  $s \in S$ , then  $s_0$  is that naximum of S and write so=max1 so is the largest element in the set  $-$  f there exists  $s_{s}$  S satisfying  $s_{s}$   $\leq$  s for all  $s \in S_1$  then  $s_0$  is the  $\frac{1}{n}$  inimum of S and write so=min( $\frac{1}{n}$ so is the smallest element in the set Def: (bounded above/below): Suppose SSF<br>for an ordered field F. for an ordered field <sup>F</sup>  $Lf$  there  $exisfs$   $N\lfloor ef$  $s \in M$   $Y$   $s \in S$ , then  $S$  is bounded above and  $M$  is an upper bound of: If there  $exists$   $m^{e}F$  $s \geq m$   $\forall s \in S$ , then S is bounded below and  $m'$  is a lower bound of  $S$ . . If S is bounded above and bounded below, then S is bounded.

Deff supremum infimum Consider an ordered field F · If SEF is bounded above and there<br>exists Mo E Satisfying... exists Mo<sup>e</sup>t Satistying.<br>a) Mols an upper bound if  $M$  is an upp a) I II. Is an upper bound as  $S$ <br>b) if M is an upper bound  $S$ <br>then  $M_o \leq m$ . we say<br>und wh we say Mo is the supremum of S<br>and write Mo=sup(s). (S) mo is the least upper bound . If SEF is bounded below and there exists moef Satisfuing... a) mo is a lower bound of S<br>a) mo is a lower bound of b) if m is a lower bound  $log S$ <br>then  $m_0 \ge m$ - then  $m_0 \ge m$ we say mo is the infimum of S<br>and write mo= inf(s). and write  $m_0 = i \sqrt{(s)}$ <br> $\pi_0$  is a greater to  $\pi_0$ mo is the greatest lowerbound

 $\frac{1 \text{ h} \text{m}}{1 \text{ T} + \text{m} \text{c} \text{x}(\text{c})}$  and  $\text{c} \text{r}$  and  $\text{c} \text{r}$  and  $\text{c} \text{r}$ i) It maxis) exists, then  $sup(S) = max(S)$  $\lim_{t\to 0} \mathcal{I}f$  min(s) exists, then  $\inf_{t\to 0}$  in(IS) = min(s)

Moral: The notion of supremum is a generalization of the notion of maximum

Elex: Creal numbers! The set of real numbers<br>is the ordered field containing Q is the ordered field containing IV.<br>with the property that every with the property that<br>nonempty subset SS.  $normpty$  subset  $SFR$  that is bounded above has <sup>a</sup> supremum "The Least Upper Bound Property of

Thm<sup>:</sup> The real numbers exists.

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\begin{array}{ll}\n\mathcal{E}x \to & \mathcal{E} = \mathbb{Q} \setminus a_0 \in \mathbb{Q} \setminus a_0 \in b \\
S = \{q \in \mathbb{Q} : a \in q \in b\} \\
\underline{\text{Claim}} \cdot \text{sup}(S) = b \\
\underline{\mathbb{Q}}\cdot \text{Equation of } S, b \text{ is an upper} \\
\overline{\text{bbund of } S} \cdot \text{Fix } \mathbb{Q} \text{ the } \mathbb{Q} \text{ s.t. } s \in \mathbb{M} \\
\text{We } S = S \quad \text{If } s \in S \text{ if } s \in \mathbb{Q} \text{ is a non-odd} \\
\text{Since } s \in \mathbb{M} \text{ and } s \in S, a \in \mathbb{M}.\n\end{array}
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$$
\begin{array}{ll}\n\text{Assume, for the sake of contradiction,} \\
\text{that } b > M. \\
\text{But, } S = \mathbb{Q} \text{ and } \mathbb{Q} \text{ is a non-odd} \\
\text{that } b > M. \\
\text{But, } S = \mathbb{Q} \text{ and } \mathbb{Q} \text{ is a non-odd} \\
\text{that } s = m \text{ and } \mathbb{Q} \text{ is a non-odd} \\
\text{and } S. \\
\text{Therefore } b \in \mathbb{M}.\n\end{array}
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\begin{array}{ll}\n\text{We conclude } b = \text{sup}(S). \\
\text{We conclude } b = \text{sup}(S).\n\end{array}
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prove on HW