Lecture 5: Highlights

Homework 1 and 2 Solutions Posted

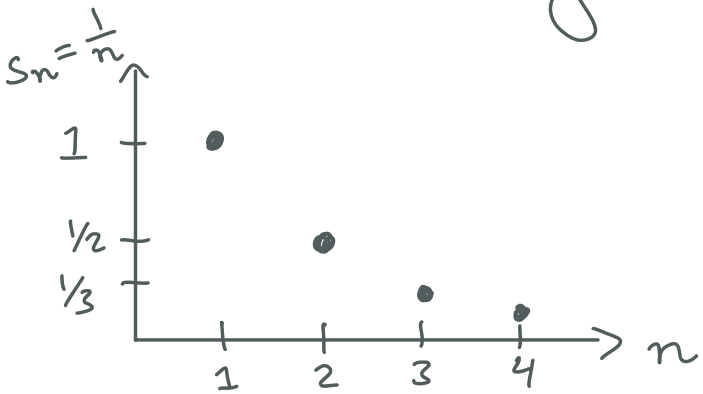
Def (sequence): A sequence is a function whose domain is a set of the form $\{m, m+1, m+2, \ldots\}$ for some $m \in \mathbb{Z}$. We will study sequences whose range is $\mathbb{R}$.

Remark:

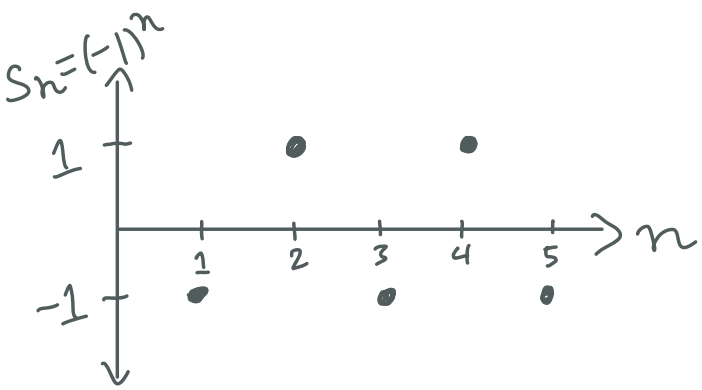
To emphasize that a sequence is special type of function...

instead of writing $f(n)$, we write s_n

$(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots)$



$(-1, 1, -1, 1, \ldots)$

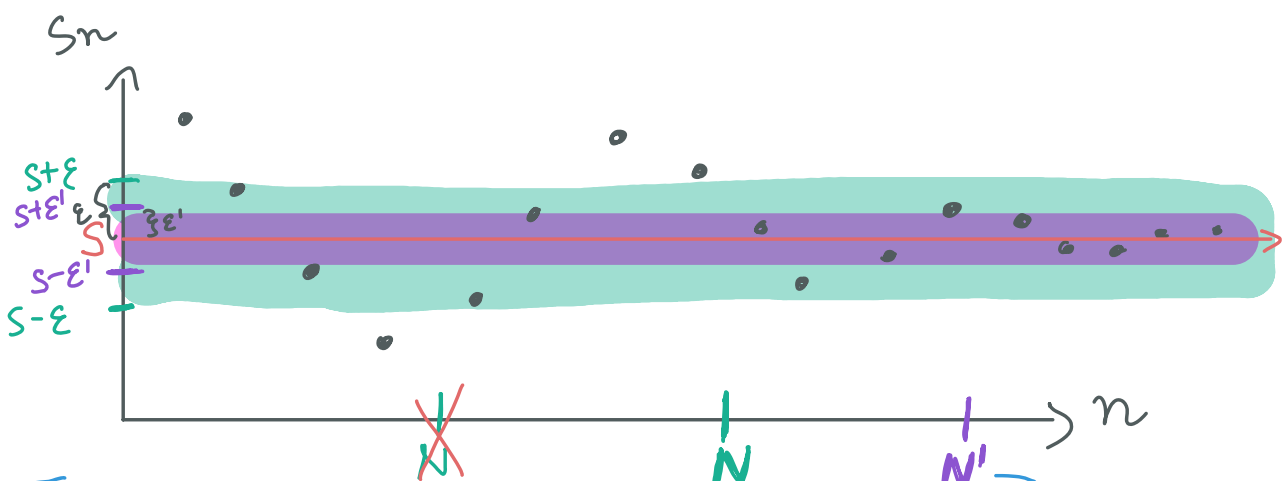


Def (convergence):

- A sequence s_n of real numbers **converges** to some $s \in \mathbb{R}$ provided that
  [for all $\varepsilon > 0$, there exists $N \in \mathbb{R}$ so that $n > N$ ensures $|s_n - s| < \varepsilon$.]

- The number s is the **limit** of s_n, and we write $\lim_{n\to\infty} s_n = s$ or $s_n \to s$.

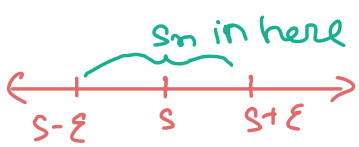
- A sequence that does not converge to any $s \in \mathbb{R}$ it is said to **diverge**.



[for all $\varepsilon > 0$, there exists $N \in \mathbb{R}$ so that $n > N$ ensures $|s_n - s| < \varepsilon$.]

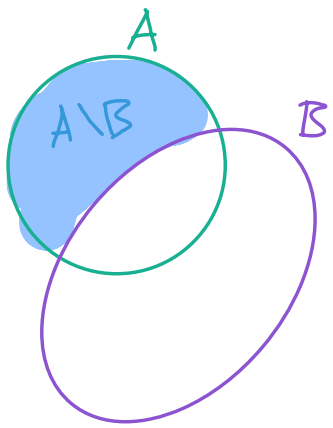
$\Updownarrow$

$s - \varepsilon < s_n < s + \varepsilon$



Recall:

- $\exists$ means "there exists"
- $\nexists$ means "there does not exist"
- $\exists!$ means "there exists unique"
- $\forall$ means "for all"
- Given sets A, B,
$$A \setminus B = \{a \in A : a \notin B\}.$$



What does $x \notin A \setminus B$ mean?

- It is possible $x \in A \cap B$
$x \in B$
$x \in A^c$

$x \notin A \setminus B \iff x \in A^c$ or $x \in B$

Want to prove theorems?
Design new algorithms?
Use math to study biology, linguistics, and AI?
Eat pizza?🍕




MATH RESEARCH

Info session for undergraduates

Wednesday, January 24th, 5-6pm
South Hall Room 6635

Come enjoy pizza and learn about undergraduate research in math at UCSB and at universities around around the world.