Math 117: Midterm 1a

Tuesday, February 6th, 2024

Name: ____

Student ID #: ____

Signature: ____

This is a closed-book and closed-note examination. Calculators are not allowed. Please show your work in the space provided. I will provide scratch paper—other forms of scratch paper are not permitted. If you continue a problem on the back of a page, please write "continued on back". Partial credit will be given for partial answers. You have 1 hour and 15 minutes.

Question	Points	Score
1	24	
2	28	
Total	52	

Question 1 (24 points)

Let A and B be nonempty bounded subsets of $\mathbb{R}$, and define $A + B = \{a + b : a \in A \text{ and } b \in B\}$.

(a) State the definition of what it means for a nonempty set $S \subseteq \mathbb{R}$ to be bounded above.

(b) Prove that, for all $b \in B$, $\sup(A + B) - b$ is an upper bound for A.

(c) Prove that $\sup(A + B) = \sup A + \sup B$.

Question 2 (28 points)

(a) State the definition of a convergent sequence.

(b) State the definition of what it means for a sequence to diverge to $+\infty$.

(c) Suppose the limit of t_n exists. Prove that

$$\lim_{n\to+\infty} |t_n| = \left| \lim_{n\to+\infty} t_n \right| .$$

(You may use the fact that $|+\infty| = +\infty$ and $|-\infty| = +\infty$.)

(d) Give an example for which $\lim_{n\to+\infty} |t_n|$ exists, but the equality from part (c) does not hold. You do not need to justify your example.