Math 117: Midterm 1b
Thursday, February 8th, 2024

Name: _____

Student ID #: _____

Signature: _____

This is a closed-book and closed-note examination. Calculators are not allowed. Please show your work in the space provided. I will provide scratch paper—other forms of scratch paper are not permitted. If you continue a problem on the back of a page, please write "continued on back". Partial credit will be given for partial answers. You have 1 hour and 15 minutes.

Question	Points	Score
3	32	
4	16	
Total	48	

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Consider a nonempty set $A \subseteq \mathbb{R}$.

(a) Suppose A is bounded below. Prove that there exists a sequence a_n, satisfying

$$\{a_n : n \in \mathbb{N}\} \subseteq A$$

and

$$\inf A \leq a_n \leq \inf A + \frac{1}{n} \text{ for all } n \in \mathbb{N}.$$

(b) Prove that the sequence you found in part (a) satisfies $\lim_{n \to \infty} a_n = \inf A$.

(c) Now suppose A is not bounded below. Prove that there exists a sequence a_n satisfying

$$\{a_n : n \in \mathbb{N}\} \subseteq A$$

and

$$a_n \leq -n \text{ for all } n \in \mathbb{N}.$$

(d) Prove that the sequence you found in part (c) satisfies $\lim_{n \to +\infty} a_n = \inf A$

Question 4 (16 points)

Lightning Round!
You do not need to show your work or justify your answers.

(1) Consider the set $S = \cup_{n=2}^{\infty} \left(\frac{1}{2}, 1\right]$.

    (i) Does the set have a maximum? What is $\sup(S)$?

    (ii) Does the set have a minimum? What is $\inf(S)$?

(2) Circle the correct answer:

    (i) Which example fails the criteria for the theorem that the limit of a quotient is the quotient of the limits?

       (a) $(1 + \frac{1}{n})/(2 - \frac{1}{n})$
       (b) $(1 + (1/2)^n)/(1/n)$
       (c) $(1/n) \cdot \sin(n\pi)/(1 + e^{-n})$
       (d) $(n^{1/n})/(1 + 3/n)$
       (e) $2^{-n}/(1 + 2^{-n})$

    (ii) Suppose $\lim_{n \to +\infty} s_n = -\infty$. What property *must* s_n satisfy?

       (a) bounded
       (b) monotone
       (c) unbounded
       (d) decreasing
       (e) limit does not exist

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