Math 117: Midterm 2a

Tuesday, March 5, 2024

Name: _____

Student ID #: _____

Signature: _____

This is a closed-book and closed-note examination. Calculators are not allowed. Please show your work in the space provided. I will provide scratch paper—other forms of scratch paper are not permitted. If you continue a problem on the back of a page, please write "continued on back". Partial credit will be given for partial answers. You have 1 hour and 15 minutes.

Question	Points	Score
1	28	
2	28	
Total	56	

Question 1 (28 points)

Suppose s_n and t_n are bounded sequences.

(a) State the theorem that the limit of the sum is the sum of the limits.

(b) Prove that, for all $N \in \mathbb{N}$,

$$\sup\{s_n + t_n : n > N\} \leq \sup\{s_n : n > N\} + \sup\{t_n : n > N\}.$$

(c) Prove that $\limsup(s_n + t_n) \leq \limsup s_n + \limsup t_n$.

(d) Give an examples of bounded sequences s_n and t_n for which $\limsup(s_n + t_n) < \limsup s_n + \limsup t_n$.

Question 2 (28 points)

Consider a sequence s_n.

(a) State the definition of $\limsup s_n$.

(b) If $\limsup |s_n| < +\infty$, prove that s_n is a bounded sequence.

(c) If s_n is a bounded sequence, prove that $\limsup |s_n| < +\infty$.

(d) If $\limsup |s_n| < +\infty$, prove that s_n has a convergence subsequence.