

MATH 117: MIDTERM 2B

Thursday, March 7th, 2024

Name: _____

Student ID #: _____

Signature: _____

This is a closed-book and closed-note examination. Calculators are not allowed. Please show your work in the space provided. I will provide scratch paper—other forms of scratch paper are not permitted. If you continue a problem on the back of a page, please write “continued on back”. Partial credit will be given for partial answers. You have 1 hour and 15 minutes.

| Question | Points | Score |
|----------|--------|-------|
| 3 | 30 | |
| 4 | 14 | |
| Total | 44 | |

Question 3 (30 points)

Define a sequence s_n as follows: $s_1 = 1$ and, for $n > 1$, $s_{n+1} = \left(\frac{n}{n+1}\right) s_n^2$.

- (a) State the theorem that the limit of a quotient is the quotient of the limits.
- (b) Prove that $0 \leq s_n \leq 1$ for all $n \in \mathbb{N}$.
- (c) Prove that s_n is a decreasing sequence.
- (d) Explain why s_n converges.
- (e) Use the definition of s_n to find the value of s , where $s = \lim_{n \rightarrow +\infty} s_n$.

In part (e), you may assume and use all the main limit theorems from class (“limit of sum is sum of limits”, “limit of product is product of limits”, “limit of quotient is quotient of limits”), without further justification. You may also use the following fact from the homework without further justification:

Fact: If c_k is a convergent sequence, then $\lim_{k \rightarrow +\infty} c_{k+1} = \lim_{k \rightarrow +\infty} c_k$.

Question 4 (14 points)

Lightning Round!

You do not need to show your work or justify your answers.

(1) Consider the sequence $a_n = \cos\left(\frac{n\pi}{4}\right)$.

(i) What is $\limsup_{n \rightarrow +\infty} a_n$ and $\liminf_{n \rightarrow +\infty} a_n$?

(ii) Does the sequence have a limit? Is it a Cauchy sequence?

(2) State whether the following statements are true or false. If they are false, provide a counterexample. You do not need to justify your counterexample.

(i) Every convergent sequence is bounded.

(ii) Every monotone sequence is Cauchy.