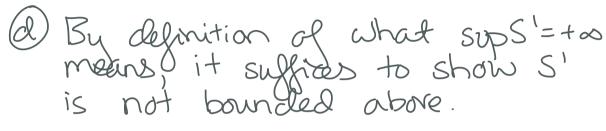
Gractice Midterm 1 Solutions () Katy Cruig, 2024 D
C) S is bounded below if there exists mo ∈ R s.t. mo ≤ s for all s ∈ S. (b) Suppose a=0 is a lower bound for S. Then ass Yses, which is equivalent to 5= a YSES Furthermore, this is equivalent to tis is equivalent to a being an upper bound for S'. © If infs >0, then by part (D, we use the fact that infs is a lower bound for S to conclude that infs is an upper bound for S! Suppose M is an upper bound for S'. Since  $S' \in (0, +\infty)$  is nonempty, M must be strictly positive. By part D, m is a lower bound for S.

Thus, by definition of the greatest lower bound,  $\overline{m} = Ghgs$ . Thus, ings = M. This shows that infs is the least upper bound of S!. Thus infs = sup S!.



Suppose, for the sake of contradiction that M were an upper bound for S! Again, since  $S' = (o_1 + \infty)$  is nonempty, we have M > 0. By part (b), m > 0. is a lower bound for S. This contradicts that infS=0 is the greatest lower bound.

Thus, S' is not bounded above.

a) A sequence sn converges to a limit SEIR IF, VE>0, JNERS.t. n7N ensures Isn-slkE.

(b) A sequence Sn doed not converge to a limit SEIR if  $\exists z > 0 s.t.$ for all NER,  $\exists n^2N s.t.$ |sn-s| ? ?.

- $\bigcirc Fix \in \mathbb{P}0. \text{ Let } N = \frac{4}{2}. \text{ Then } n^{2}N \text{ ensures}$   $\frac{4}{n} < \varepsilon \iff \frac{4n}{n^{2}} < \varepsilon \iff \frac{n+3n}{n^{2}} < \varepsilon \implies \frac{n+3}{n^{2}} < \varepsilon \implies 0$ 
  - $\frac{|n-3|}{n^2} < \varepsilon \Longrightarrow \left| \frac{n-3}{n^2} \right| < \varepsilon \Longrightarrow \left| \frac{n-3}{n^2+q} \right| < \varepsilon \longleftrightarrow \left| \frac{n-3}{n^{2+q}} 0 \right| < \varepsilon.$
  - Since 2>0 was arbitrary, this gives the result.

(2) Assume, for the sake of contradiction, that sn converges to some SER. Then, for E=1, there exists NER so that n>N ensures

$$|s_n - s| < | \iff s - | < s_n < s + |$$
  
 $\iff s - | < (n + 1)^2 - 2 < s + |$   
 $\iff s + 1 < (n + 1)^2 < s + 3$ 

By the lemma following the Archimedean Property,  $\exists m \in |N|$  so that  $m^{2}s+3$ . Let k = max(m, N+1). Then  $k \ge m^{2}s+3$ and  $k \ge N$ . The latter ensured:



This contradicts B. Thus Sn must not converge to any SER. B C  $\rule{C}$   $\rule{C}$  $\rule{C$ 

6 C) Fix nEIN. Since A is not bounded above, n is not an upper bound for A, so there exists aneA s.t. an?n. In this way, we know there exists a sequence an with ant A and an m for all nEIN.

(d) Fix M=0. Let N=M. Then for all n=N, an=n=N=M. Thus an diverges to to. This shows himson=sopA

(1) (i) no, 
$$\sup(c) = +\infty$$
  
(ii) yes,  $iny(c) = 0$   
(z) @

