Math 201a: Midterm 2

Thursday, November 12, 2026

Name: _____

Student ID #: _____

Signature: _____

This is a closed-book and closed-note examination. Please show your work in the space provided. You may use scratch paper. You have 1 hour and 15 minutes.

Question	Points	Score
1	8	
2	10	
3	12	
Total	30	

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Question 1 (8 points)

Compute the following limit and justify your calculation:

$$\lim_{n\to\infty} \int_{[0,1]} \frac{1+nx^2}{(1+x^2)^n}\, d\lambda(x).$$

You do not need to explain why the functions are measurable.

Hint: you may use Bernoulli's inequality: $(1+t)^n \geq 1+nt$ *for* $t \geq -1$ *and* $n \in \mathbb{N}$.

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Question 2 (10 points)

Suppose $(X, \mathcal{M}, \mu)$ is a measure space and f_n, f are $(\mathcal{M}, \mathcal{B}_{\bar{\mathbb{R}}})$-measurable functions satisfying $f_n(x) \to f(x)$ for μ-a.e. $x \in X$. Consider $S \in \mathcal{M}$ arbitrary.

(a) Suppose $f_n \geq 0$ μ-a.e. for all $n \in \mathbb{N}$. Prove that

$$\lim_{n\to+\infty} \int f_n d\mu = \int f d\mu < +\infty \implies \lim_{n\to+\infty} \int_S f_n d\mu = \int_S f d\mu, \ \forall S \in \mathcal{M}.$$

(b) Does the result continue to hold without the nonnegativity assumption on f_n? Justify your answer with a proof or counterexample.

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Question 3 (12 points)

Consider a measure space $(X, \mathcal{M}, \mu)$ and measurable functions $f_n, f : X \to \mathbb{R}$.

(a) Suppose $\mu(X) < +\infty$ and $f_n \to f$ μ-almost everywhere. Prove that $f_n \to f$ in measure.
Hint: for a fixed $\epsilon > 0$, consider the sets $E_N = \bigcup_{n \geq N} \{x : |f_n(x) - f(x)| \geq \epsilon\}$.

(b) Give a counterexample to show that the result of part (a) does not hold if $\mu(X) = +\infty$.

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