Homework 3 Solutions

(7)

(i) Fix $\varepsilon > 0$ arbitrary. Choose $N \in \mathbb{N}$ so that $N > \frac{1}{\varepsilon} + 2$, which is possible via the Archimedean Property. Then $n \geq N$ ensures

$$\left|\frac{1}{n-2} - 0\right| = \left|\frac{1}{n-2}\right| = \frac{1}{n-2} < \varepsilon.$$

$n \geq N > 2$

$n \geq N > \frac{1}{\varepsilon} + 2$
$\Leftrightarrow n - 2 > \frac{1}{\varepsilon}$
$\Leftrightarrow \varepsilon > \frac{1}{n-2}$

Since $\varepsilon > 0$ was arbitrary, this gives the result.

(ii) Fix $\varepsilon > 0$. Choose $N \in \mathbb{N}$ s.t. $N > \frac{4}{\varepsilon}$. Then $n \geq N$ ensures $\varepsilon > \frac{4}{n}$, so

$$\left|\frac{2n}{n+2} - 2\right| = \left|\frac{2n - 2(n+2)}{n+2}\right| = \left|\frac{-4}{n+2}\right| = \frac{4}{n+2} < \varepsilon.$$

Since $\varepsilon > 0$ was arbitrary, this gives the result.

(iii) Fix $\varepsilon > 0$. Choose $N \in \mathbb{N}$ s.t. $N > \frac{1}{\varepsilon}$.
Then $n \geq N$ ensures $\varepsilon > \frac{1}{n}$, so

$$\left| \frac{(-1)^n}{n} - 0 \right| = \left| \frac{(-1)^n}{n} \right| = \frac{1}{n} < \varepsilon .$$

Since $\varepsilon > 0$ was arbitrary, this gives the result.