Math CCS 117: Practice Final Exam
(Not to be turned in)

Question 1

Let $f : [a, b] \to \mathbb{R}$ be an increasing function; that is, for all $x, y \in [a, b]$, $x < y$ implies $f(x) \leq f(y)$. Prove that f has a right limit at every $c \in [a, b)$ and a left limit at every $c \in (a, b]$.

Question 2

Consider a sequence $s_n : \mathbb{N} \to \mathbb{R}$. Prove that $\limsup |s_n| = 0$ if and only if $\lim s_n = 0$.

Question 3

Let f be defined on $[0, 1]$ by the formula

$$f(x) = \begin{cases} \frac{1}{n} & \text{if } x \in \mathbb{Q} \text{ and } x = \frac{m}{n} \text{ is in lowest terms, } m, n \in \mathbb{N}. \\ 0 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x = 0. \end{cases}$$

Prove that f is continuous only at the irrational points of $[0, 1]$.

Question 4 - Extra Credit

Let f be an increasing function on $[a, b]$. Prove that the set of points at which f is not continuous is countable.