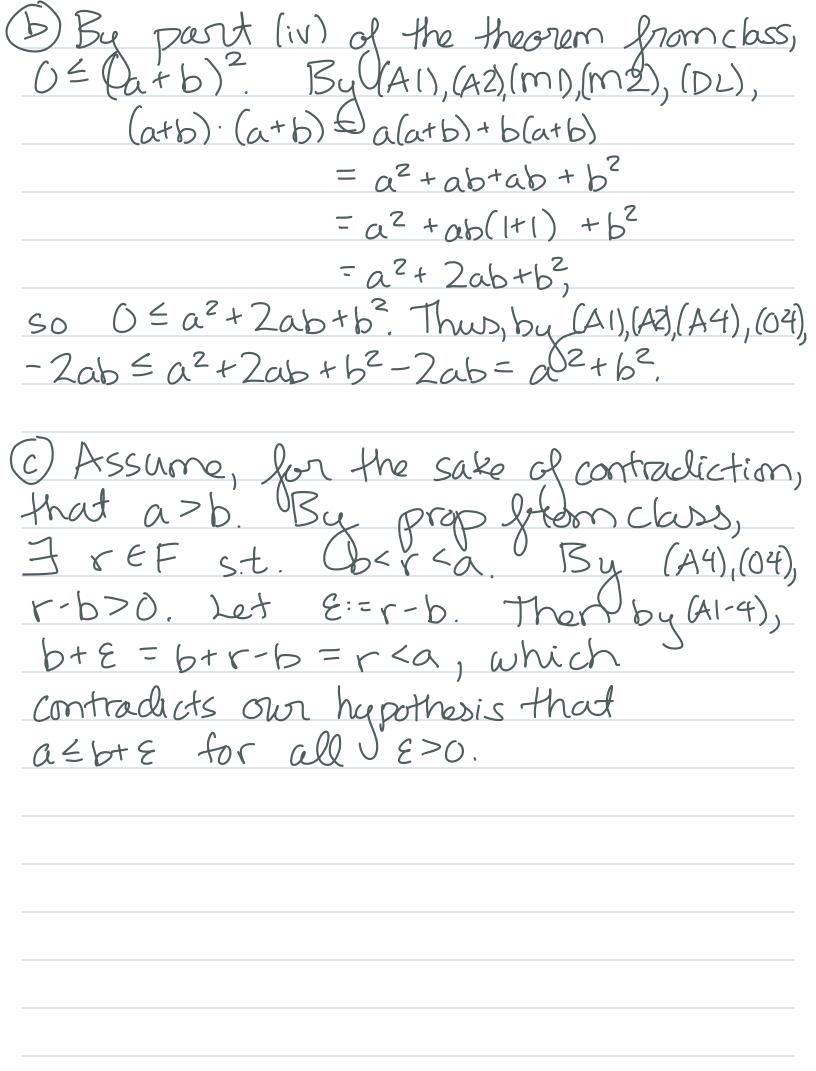
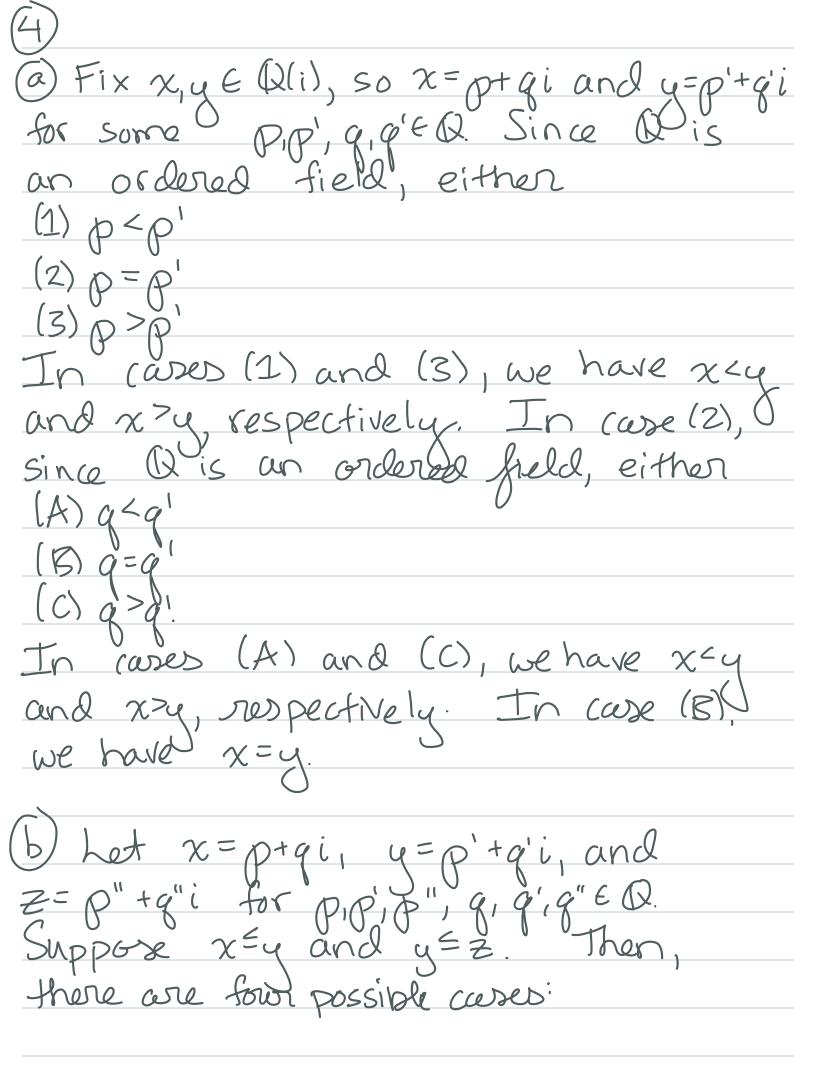
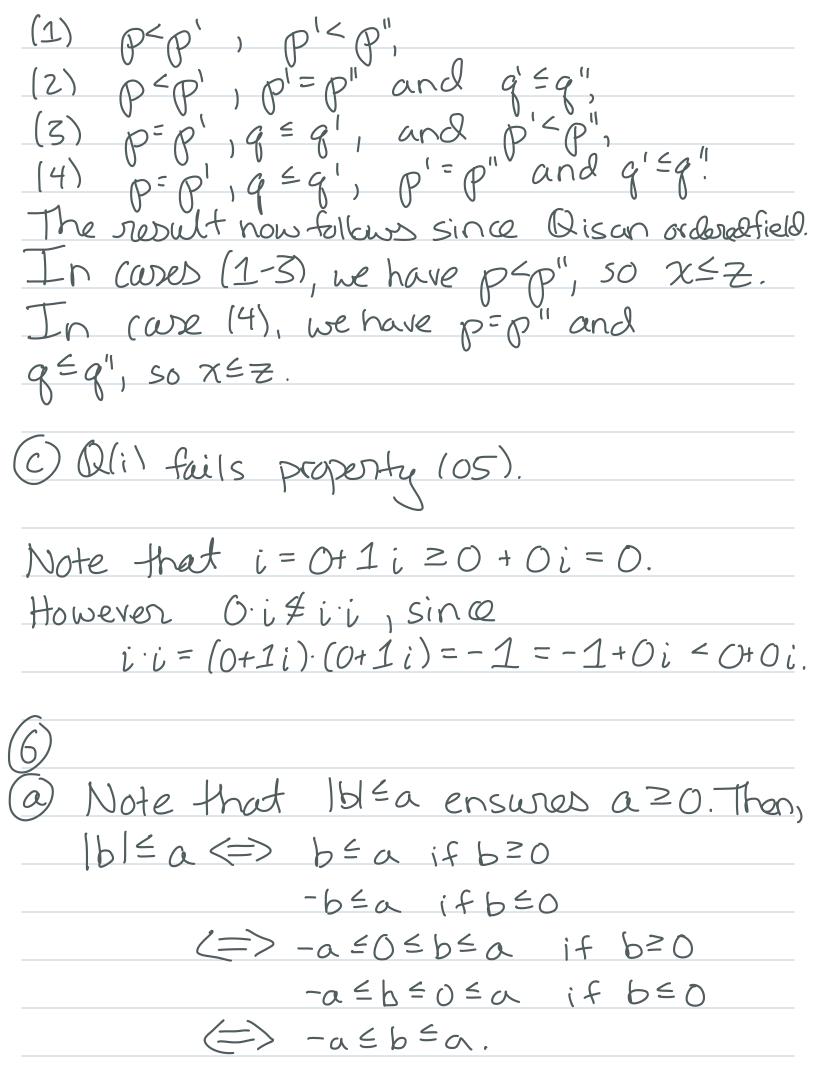
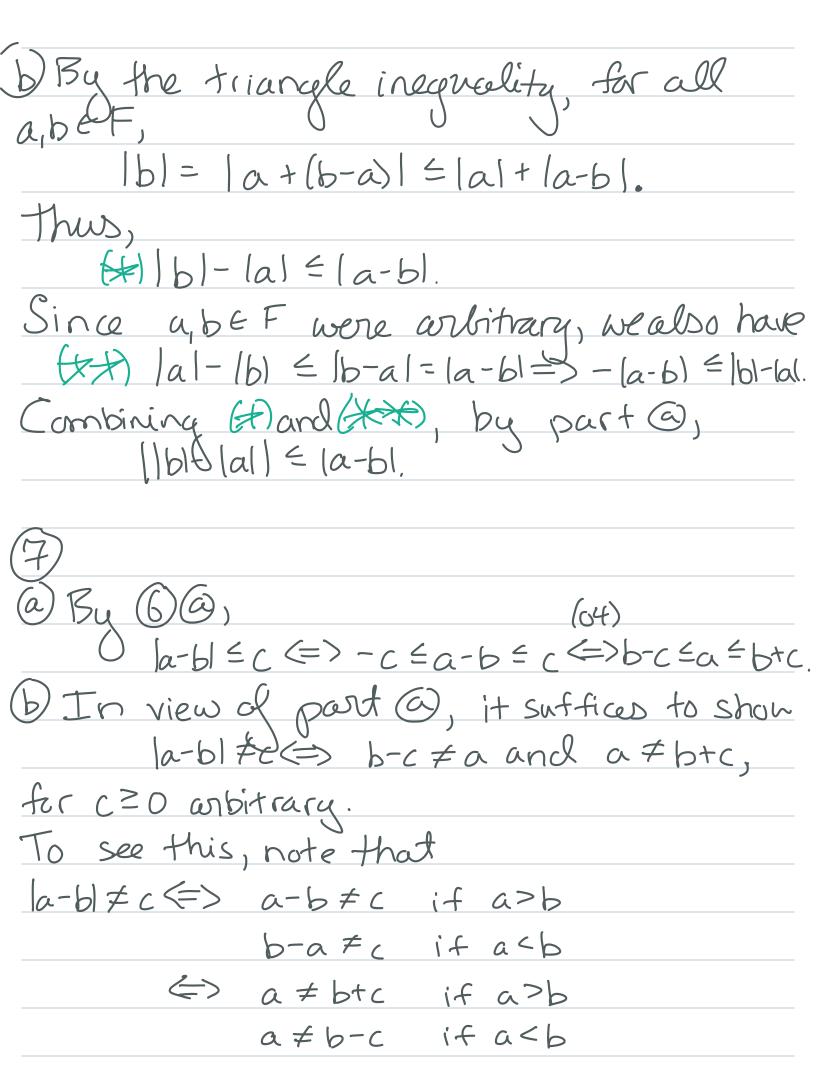
Homework 1 Solutions
CS 117, S25
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0 7.
(2)
By part (v) of Throforn class, OKE, OKE
so it sullices to show b< a.
By part (v) of Throforn class, $0 < b$, $0 < a$, so it suffices to show $b < a$. Note that $a < b = > a \le b$
(05) a. (a.b) \(b\) (\(\alpha\) \(\bar{b}\) \(\bar{b}
=> (a·a)·b = (b·b)·a => 1·b = 1·a
=> \frac{1}{5} \fr
It remains to show $b \neq a$. Suppose for the sake of contradiction that $b = a$. Then $b = a = b$ (ab) = $a(a \cdot b)$ (m2, m3) Then $b = a = a \cdot a \cdot b = b$ (m4) $a = a \cdot a \cdot b = b \cdot a = b$,
for the sake of contradiction that == a.
Then, $b = a = b$ (ab) = a(a·b) (mz, ms)
=> (b.pl.a=(a.al.p =>1.a=1.p=) a=b,
which contradicts that a < b. Therefore
b +a.

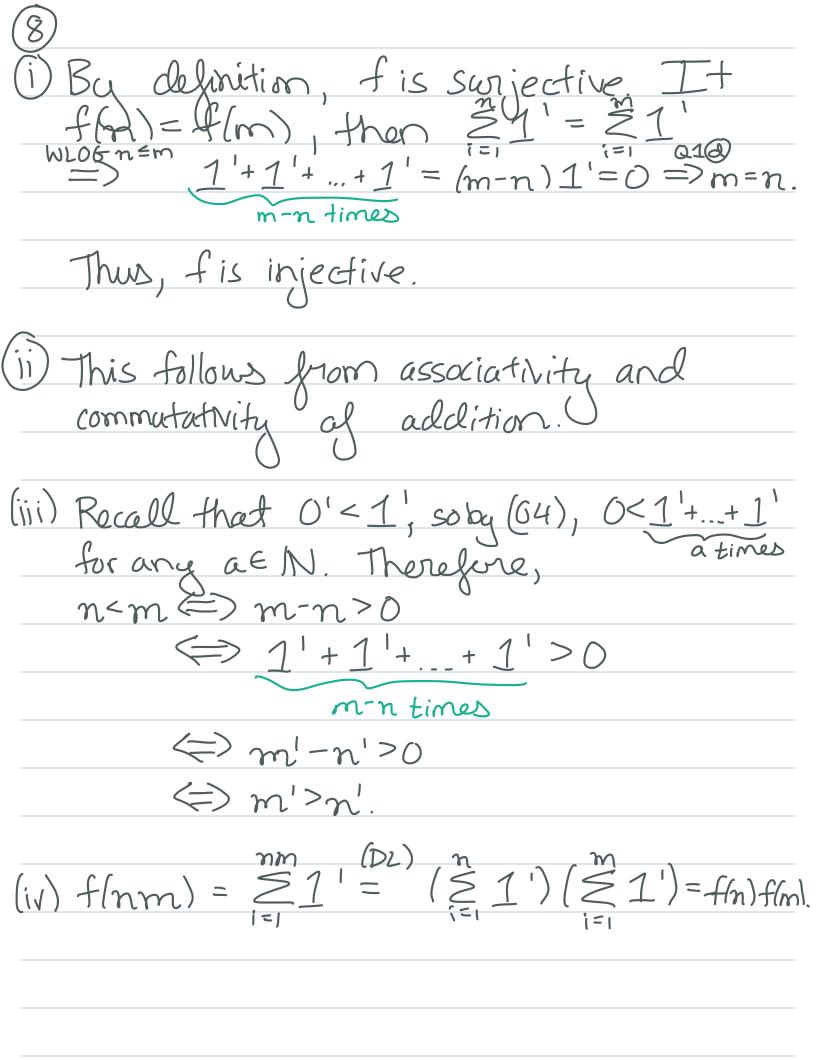








	⟨= ⟩	a + b+c	and c	2 + b-c	
		since c >0, to possible if a	his is only	Since (>0, -	this is f a < b.
8					
[insert]	<u> </u>				



Let $g(a) = (f(a))$ if $a \in IN$ O' if $a = O(-f(-a)) if -a \in IN.$
$0 \begin{cases} 0 \\ \text{if } \alpha = 0 \end{cases}$
(-f(-a) if -a = 1N.
In the previous solution, we showed that
In the previous solution, we showed that $\forall a \in IN'$, $a > 0$. Thus $\forall b \in IN'$, $b < 0$.
Since fill = 1811 is curiostive
Since f: IN > IN' is swijective,
a is surjective. It gla) = a(b), then either
Q is Surjective. If $g(a) = g(b)$, then either $(A) g(a) = g(b) > 0' = g(a) = f(a) = f(b) = g(b)$
=> a=b, by injectivity of f
$\Rightarrow a=b, by injectivity of f$ $(B) g(a)=g(b)<0' \Rightarrow f(-a)=f(-b)$ $\Rightarrow -a=-b, by injectivity of f$
$\Rightarrow -\alpha = -b. \text{ by injectivity of } f$
$= -a = -b, by injectivity of f$ $\Rightarrow a = b$
(C) $a(a) = a(b) = 0 \Rightarrow a = b = 0$, by defining. Thus a is injective.
Thus a is injective.
J VWS CL IS IT GEST C.
To see (ii), note that Ya, b & Z,
Case 1: either a=0 or b=0. WLOE, suppose b=0, so
a(a)+a(b)=a(a)+0'=a(a+o) (ax 2: a>0 and b>0. The result follows fram (x8)

Case 3: aso and bso.

g(a)+g(b) = -f(-a)-f(-b) = -f(-a-b) = g(a+b) (are 4: a and b have opposite signs. WLO6 a>0, b<0. Then, $g(a)+g(b)=f(a)-f(-b)$. If $a>-b$, let $m=a+b$, $n=-b$, so $f(a)-f(-b)=f(m+n)-f(n)=f(m)=g(a+b)$ If $-b>a$, let $m=-b-a$, $n=a$, so $f(a)-f(-b)=f(n)-f(m+n)=-f(m)=g(a+b)$ This shows g satisfies (ii). To see (iii), note that $a>b-a>0$ $(a)=-g(a)=-g(b)-a>0$ $(a)=-g(a)=-g(a)>0$ $(a)=-g(a)<-g(b)$
If $-b>a$, let $m=-b-a$, $m=a$, so $f(a)-f(-b)=f(n)-f(m+n)=-f(m)=g(a+b)$. This shows a satisfies (ii). To see (iii), note that
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This shows a satisfies (ii). To see (iii), note that
This shows a satisfies (ii). To see (iii), note that
To see (iii), note that $a < b < > b - a > 0$ $(a < b < > b - a > 0)$ $(a < b < > a < b < -a < a < a < a < a < a < a < a < a < $
a < b < b > b-a>0 $(ii) a(b)+a(-a)=a(b-a)=f(b-a)>0$ $g(a)=-g(a)b)defof(a)$ $a(b)-a(a)>0$ $2=a(a) < a(b).$
$ \frac{(a)}{(a)} = \frac{a(b)}{a(a)} + \frac{a(-a)}{a(a)} = \frac{a(b-a)}{a(b)} = \frac{a(b)}{a(a)} = \frac{a(b)}{a(b)} = \frac{a(b)}{a(a)} = \frac{a(b)}{a($
2= $a(b)-a(a)-02=$ $a(a)< a(b)$.
$\sim \gamma q(\alpha) - q(0)$.
Finally, to see (iv), for a, b = Z,
(use Deither a=0 or b=0. By (ii), we see a(0)=0;
Finally, to see (iv), for a,bt Z, (axe 1) either a=0 or b=0. By (ii), we see $g(0)=0$; So $g(ab) = g(0) = 0' = g(a)g(b)$.

Case 2: a and	bhave same	Sign	
Then	a (ab) = f(ab)	=ffa)f(b)	if a,b>0
Case 2: a and Then	O)f(-a)f(-b)	ifa,b<0
		= g(a) g(b)	
Case 3: a and	b have opp	osite sign	S. WLO6a70.
	g(ab) = -f(al-b)		