Math CCS 117: Midterm 1

Tuesday, April 29, 2025

Name: ______________________________

Signature: ______________________________

This is a closed-book and closed-note examination. Please show your work in the space provided. You may use scratch paper. You have 1 hour and 15 minutes.

Question	Points	Score
1	10	
2	10	
3	10	
4	extra credit	
Total	30	

Question 1 (10 points)

Consider a sequence s_n satisfying $s_n \neq 0$ for all n and for which $\left|\frac{s_{n+1}}{s_n}\right|$ converges to L. If $L < 1$, show that $\lim_{n\to+\infty} s_n = 0$.

(If you use proof by induction, please carefully explain your argument.)

Question 2 (10 points)

Suppose $\lim_{n\to+\infty} s_n t_n > 0$. If $\lim_{n\to+\infty} t_n > 0$, prove that $s_n \leq 0$ for at most finitely many $n \in \mathbb{N}$—in other words, prove that the set $\{n \in \mathbb{N} : s_n \leq 0\}$ has finitely many elements.

Question 3 (10 points)

Suppose A and B are nonempty bounded subsets of $\mathbb{R}$. Define $A + B = \{a + b : a \in A \text{ and } b \in B\}$. Prove $\sup(A + B) = \sup A + \sup B$.

Question 4 - Extra Credit

Given a sequence s_n of real numbers, define its arithmetic mean by

$$\sigma_n = \frac{s_1 + s_2 + \cdots + s_n}{n}.$$

(a) If s_n converges, prove that σ_n converges.

(b) Give an example to show that the converse of part (a) is not true.

(c) Let $a_n = s_{n+1} - s_n$. Assume that $\lim_{k \to +\infty} k a_k = 0$ and σ_n converges. Prove that s_n converges.

Hint: First, show that

$$s_{n+1} - \sigma_{n+1} = \frac{1}{n+1} \sum_{k=1}^{n} k a_k.$$

Moral of the problem: while the convergence of σ_n is not, in general, sufficient to imply the convergence of s_n, if we also know that the increments of s_n converge to zero sufficiently quickly, it is sufficient.

