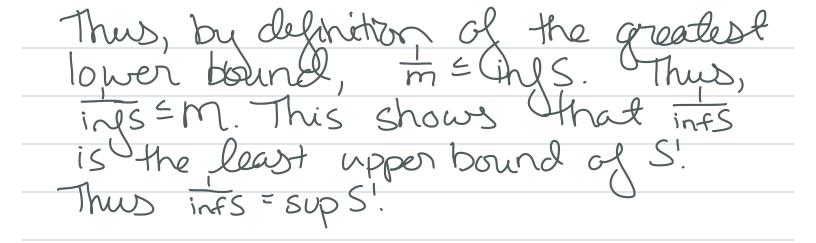
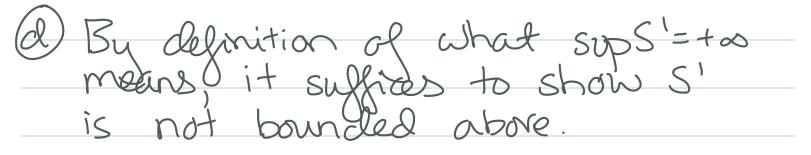
Practice Midterm 1 Solutions

<u>CS 117, S25</u> C Katy Craig, 2025 Pf: For all nEIN, sup(A)-n < sup(A). Since sob(A) is the least upper bound of A, sup(A)-n cannot be an upper bound for A. That is, I and A s.t. Sup(A)-nean. Since anEA, Jorall nEN, wehave supA-n≤an≤supA. Fix $\varepsilon > 0$. By the Archimedean theorem, $\exists N \in \mathbb{R} \text{ s.t.} 0 \quad \forall < \varepsilon. \text{ Thus, } n > N$ $ensured \quad \forall < \varepsilon, so$ SUPA-E = SUPA- n = an = SUPA and lan-supAl<E. This shows lim an=sup A.

(C) Fix nEIN. Since A is not bounded above, n is not an upper bound for A, so there exists anEA s.t. an?n. In this way, we know there exists a sequence an with an EA and an ²n for all nEIN.

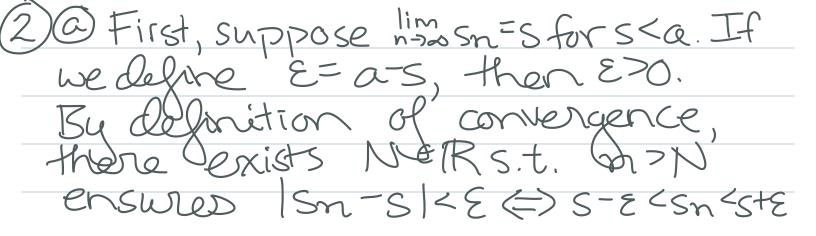
d) Fix M>O. Let N=M. Then for all n > N, an > n > N = M. Thus an diverges to + or. This shows no an = sopA

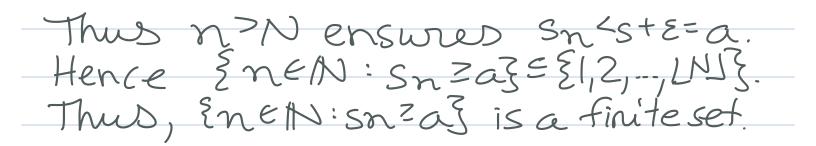




Suppose, for the sake of contradiction, that M were an upper bound for S! Again, since $S' \leq (o_1 + \infty)$ is nonempty, we have M > 0. By part (b), m > 0. is a lower bound for S. This contradicts that infS=0 is the greatest lower bound.

Thus, S' is not bounded above.





Next, suppose h=>00 Sn=-00. Let M=min{-1, a}. Then J Ns.t. sn<M≤a for all n=N. Thus, Enervisor as is a finite set.

(b) First, suppose hims tn = 2 > 0. Then n=otn > = Applying part @ with Sn=-tn and a= = we obtain that $\frac{2}{2}n\epsilon N \cdot sn = a_{s}^{2} = \frac{2}{2}n\epsilon N \cdot -tn = \frac{1}{2}$ $= \frac{2}{2}n\epsilon N \cdot tn = \frac{1}{2}\frac{2}{2}n\epsilon N \cdot tn < \frac{1}{2}$ are all phite sets. This shows the result for b= =.

Now, suppose how to =+00. Then 3 NS.t. $\forall n \geq N$, $tn \geq 1$. Thus, $\forall n \in N: tn \leq 1$? $\geq \forall n \in N: tn \leq 1$? is finite. This shows the result for b=1(3)Fix E>O. Since lim azn= a=lim azn+1) 3 Ne, No ER s.t. $n > N_e = > |a_{2n} - a| < \epsilon$ $n > N_0 = > |a_{2nt}| - a| < \epsilon$. Define N = 2: max ?Ne, Noz. Suppose $n^{9}N$. If n is even, n=2m for $m \in IN$ and $n > N \ge 2Ne => 2m \ge 2Ne$ =>m=Ne, so lan-al=lazm-al<E. OTOH, if n is odd, n = 2m - 1 for $m \in IN$, and n >N = 2No-1=> 2m-1=2No-1=>m=15 So an-al = lazm-1-al < E. Thus, in both cases, n=N ensures lan-alce. This

shows n-700 an = q.

We begin by showing Szn is decreasing and Szn-1 is increasing.

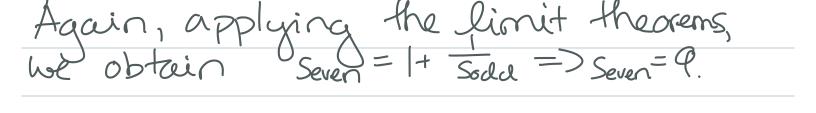
We proceed by induction. For the base dase, note that $s_1 = 1$, $s_2 = 2$, $s_3 = \frac{3}{2}$, $s_4 = \frac{5}{3}$. Suppose S2K= S2R+11 and S2K-1=S2K+1. Then $S_{2(k+2)} = [+\frac{1}{S_{2k+1}} \le [+\frac{1}{S_{2k-1}} = S_{2k}]$ So $S_{2k+3} = [+\frac{1}{S_{2(k+2)}} \ge [+\frac{1}{S_{2k}} = S_{2k+1}]$.

Now, we show ISSnE 2 UnE/N. Since Son is decreasing and So=2, we have Szn = 2 VGA. Since Szn-1 is increasing and sz=1, we have Sen-1 = 1 $\forall n$ Furthermore, Sen = 1 + $\frac{1}{S_{2n+1}} = 1$ $\forall n$ and Sen = 1 + $\frac{1}{S_{2n+1}} = 1$ $\forall n$ and Sen = 1 + $\frac{1}{S_{2n+1}} = 1$ $\forall n$ and Sen = 1 + $\frac{1}{S_{2n}} = 2$. This shows 1=Sn=2 VnEIN.

Since I= sn = 2 for all n, the subsequence of even terms and the subsequence of odd terms are both bounded and monstone. Hence, they both converge. Let lim SZK = Seven and know SZK-1 = Sodd.

Note that: $S_{2k+1} = | + \frac{1}{S_{2k}} = | + \frac{1}{1 + \frac{1}{S_{2k+1}}}$ Since Szk-121, Soda21. Thus, applying the limit theorems (quotient, sum), we have $S_{0,0,0} = \lim_{k \to \infty} S_{2k+1} = 1 + \frac{1}{1 + \frac{1}{5}}$

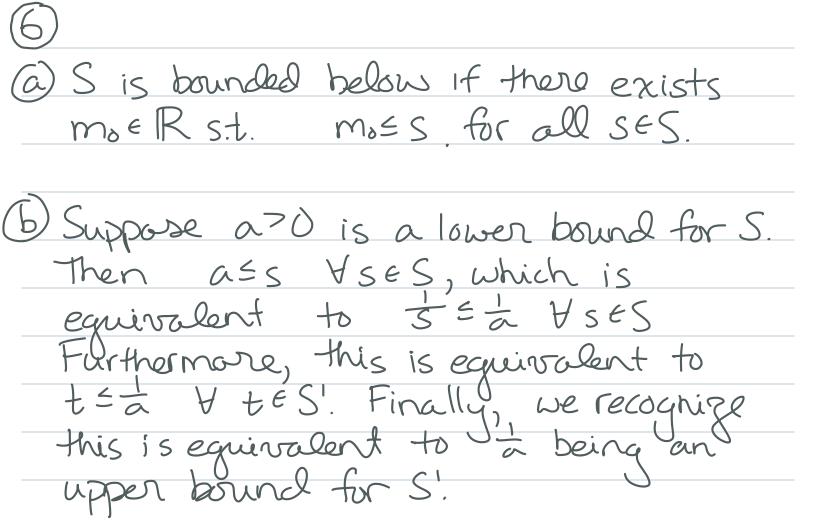
Thus, sode solves $(S_{odd} - 1) = (1 + \frac{1}{S_{odd}})^{-1}$ $(S_{odd} - 1)(1 + \frac{1}{S_{odd}}) = | \neq S_{odd} + | - 1 + \frac{1}{S_{odd}} = 1$ (=) Soão - Sooo - I = O. By the quadratic formula and the fact that sold E[1,2], we obtain Soda= P. Finally, $S_{2k} = 1 + \frac{1}{S_{2k-1}}$



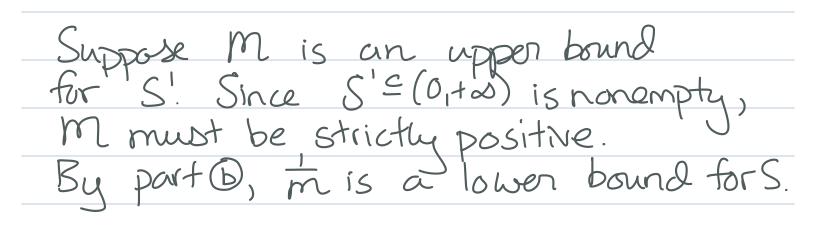
The result then follows by question 3. (5) We will show imagen=0. Since lim any =0, 7 Nos-L. n>N. ensured $|anti (< \frac{1}{2} =) |anti (< \frac{1}{2} |anti.)|$ We claim that, YN, m EIN with N>Ng Land < (=)^m lant. We have just shown the buse case m=1.0 Suppose $|a_{N+m}| < (\frac{1}{2})^m |a_N|$. By (M), $|a_{N+m+1}| < \frac{1}{2} |a_{N+m}| < (\frac{1}{2})^{m+1} |a_N|$. This proves the claim.

Since $\Xi \in (0, 1)$, by HWB Q5, $\lim_{n \to \infty} (\Xi)^m$ so $\forall N \in (N)$ with N>No, $\lim_{n \to \infty} (\Xi)^m |a_N| = 0$ by the theorem that the limit of the product is the product of the limits.

Fix EZO. Then 3 M s.t. m > M ensured (=) mlanot 1<E. Thus for n > No+M+2, we may express n = N+m for N>No+1 and m>M, so ** ensured lan [=[an1m] < (2)]and [= therefore, n-300 an=0.



© If infs >0, then by part(D, we use the fact that infs is a lower bound for S to conclude that infs is an upper bound for S!



Thus, by definition of the greatest lower bound, $\overline{m} \leq infs$. Thus, $infs \leq M$. This shows that \overline{infs} is the least upper bound of S'. Thus $\overline{infs} \equiv sup S'$. (d) By definition of what sups'= + as means, it suffices to show S' is not bounded above.

Suppose, for the sake of contradiction, that M were an upper bound for S! Again, since $S \leq (0, +\infty)$ is nonempty, we have M > 0. By part (b), m > 0. is a lower bound for S. This contradicts that infS=0 is the greatest lower bound.

Thus, S' is not bounded above.

(7) The correct definition is (2).

(a) Consider Sn = (1, -1, 1, -1, ...), s = 0. Then for $\varepsilon = 2$ and all NER, n^2N ensures $|Sn - s| < \varepsilon$.

(b) Consider sn= n, s=0. For E=0 there is no NER so that n N ensures Isn-01<E.

Consider sn=n, s=0. For E=4, Isn-sl<E is not true for all n=N.

