Math CCS 117: Midterm 2

Wednesday, May 27th, 2026

Name: _____

Signature: _____

This is a closed-book and closed-note examination. Please show your work in the space provided. You may use scratch paper. You have 1 hour and 15 minutes.

Question	Points	Score
1	10	
2	10	
3	10	
4	extra credit	
Total	30	

Question 1 (10 points)

In this problem, we will consider sequences s_n satisfying the following property, for fixed $s \in \{\pm\infty\}$:

$$\text{every subsequence } s_{n_k} \text{ of } s_n \text{ has a further subsequence } s_{n_{k_l}} \text{ s.t. } \lim_{l\to+\infty} s_{n_{k_l}} = s. \qquad (*)$$

(a) Prove that if $\lim s_n = s$, then property (*) holds.

(b) Prove that if property (*) holds, then $\lim s_n = s$.

On the homework, you proved this result for $s \in \mathbb{R}$.

Question 2 (10 points)

Suppose s_n and t_n are bounded sequences.

(a) Prove that $\liminf s_n + t_n \geq \liminf s_n + \liminf t_n$.

(b) Give an examples of sequences s_n and t_n for which $\liminf s_n + t_n > \liminf s_n + \liminf t_n$.

Question 3 (10 points)

A function $f : \mathbb{R}^d \to \mathbb{R}$ is *positively homogeneous* if, for every $\lambda > 0$, $f(\lambda x) = \lambda f(x)$. Prove that a positively homogeneous function $f : \mathbb{R}^d \to \mathbb{R}$ is convex if and only if

$$f(x + y) \leq f(x) + f(y), \quad \forall x, y \in \mathbb{R}^d.$$

Question 4 - Extra Credit

Given an interval $I \subseteq \mathbb{R}$, a function $f : I \to \mathbb{R}$ is *locally Lipschitz* if for all $a, b \in I$, $a < b$, there exists $L_{a,b} > 0$ so that

$$|f(x) - f(y)| \leq L_{a,b}|x - y|, \forall x, y \in (a, b).$$

If f is convex, recall that, for all $p, q, r \in D(f)$ with $p < q < r$, we have

$$\frac{f(q) - f(p)}{q - p} \leq \frac{f(r) - f(p)}{r - p} \leq \frac{f(r) - f(q)}{r - q}.$$

Suppose $f : \mathbb{R} \to \mathbb{R} \cup \{+\infty\}$ is convex. For any open interval $(p, q) \subseteq D(f)$, prove that f is locally Lipschitz on (p, q).