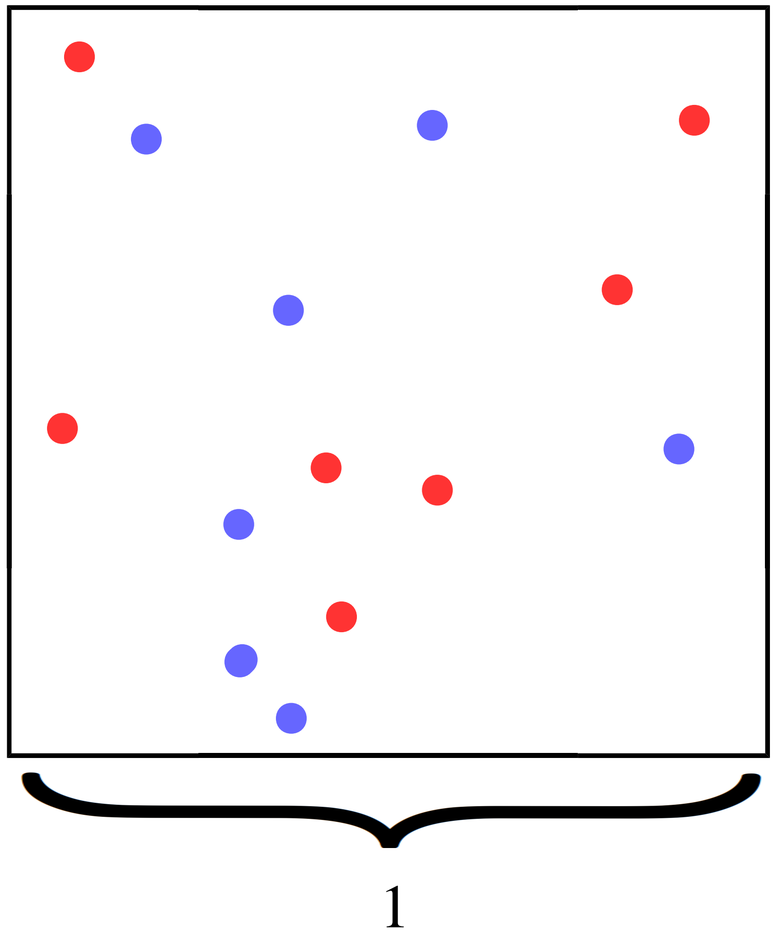
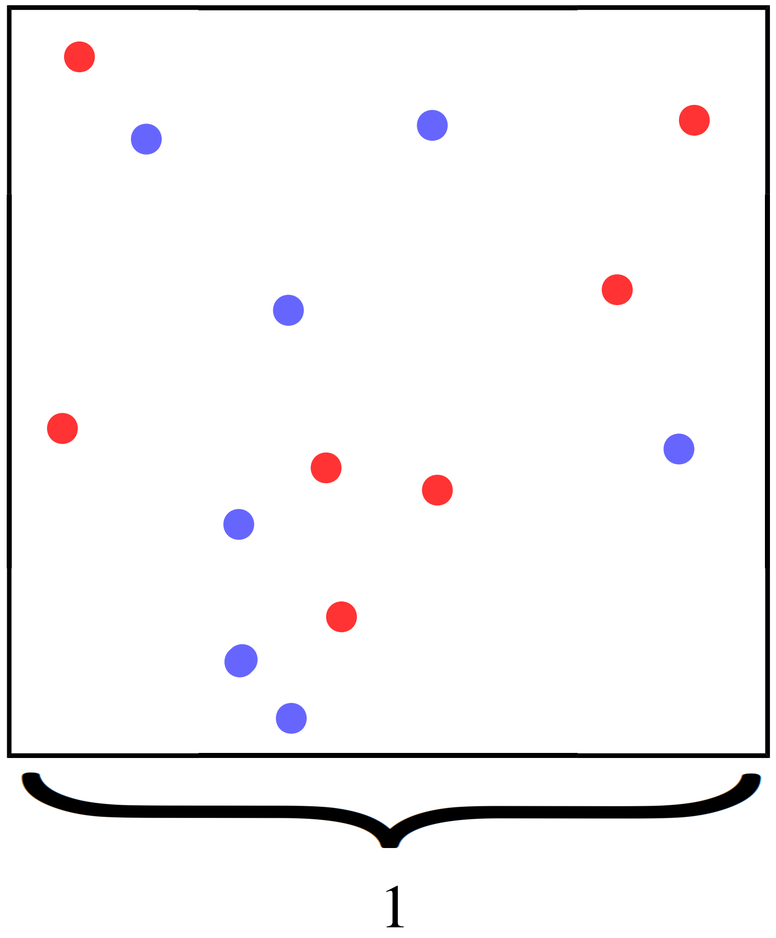
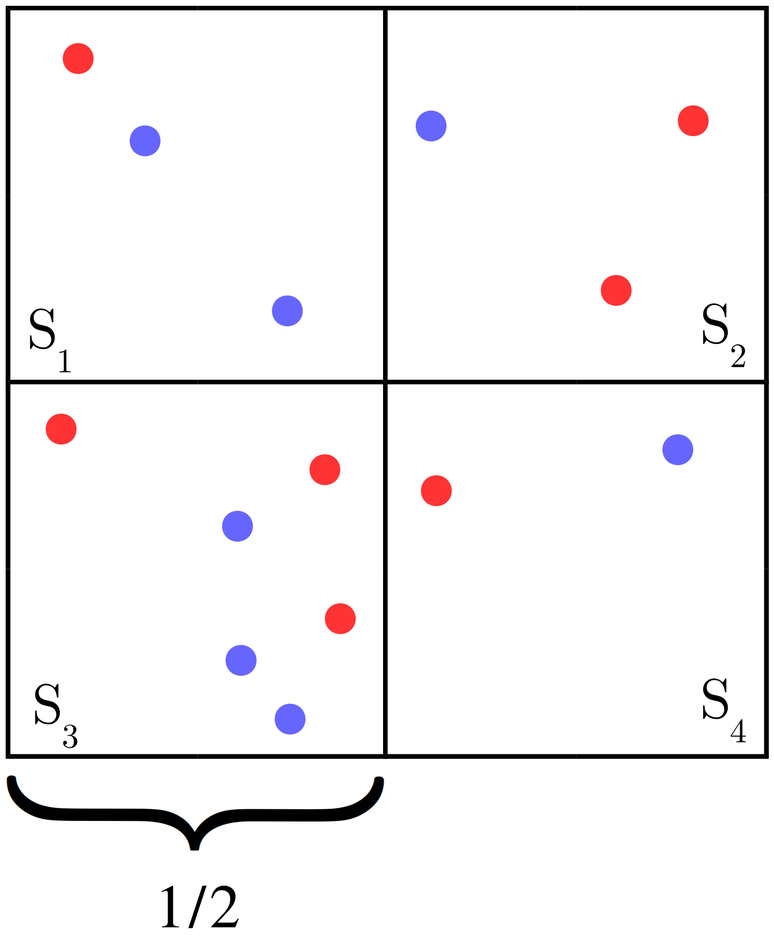
$$\mu = \frac{1}{n}\sum_{i=1}^{n}\delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n}\delta_{y_i}$$





$$\mu = \frac{1}{n}\sum_{i=1}^{n}\delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n}\delta_{y_i}$$

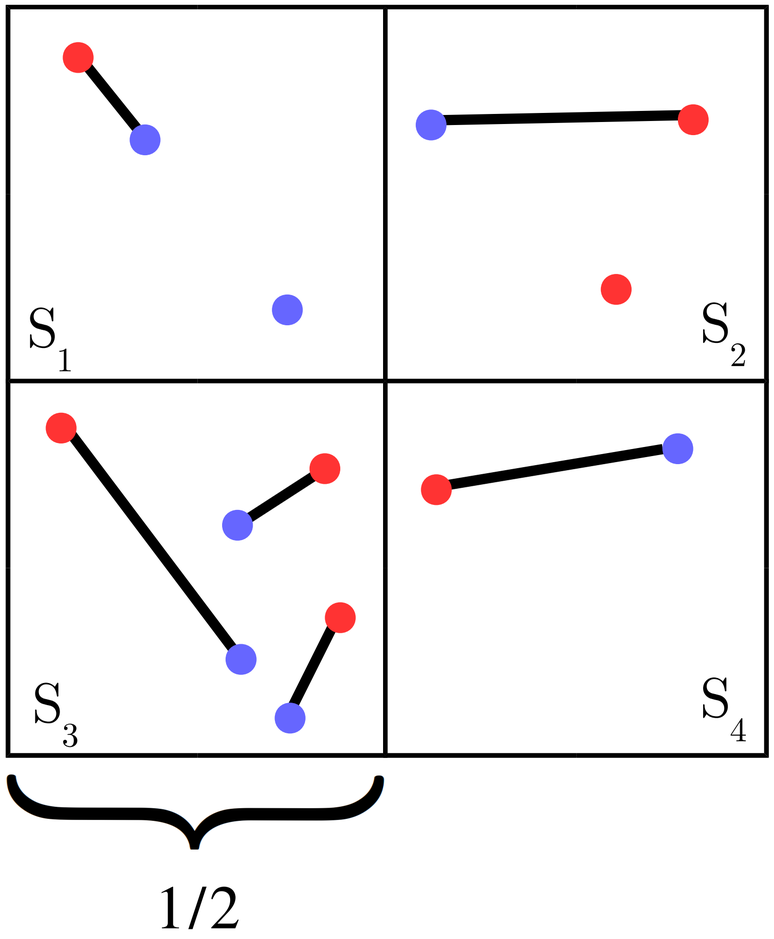
$$W_1(\mu,\nu) \leq \sqrt{d}$$



$$\mu = \frac{1}{n}\sum_{i=1}^{n}\delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n}\delta_{y_i}$$

$$W_1(\mu,\nu) \leq \sqrt{d}$$

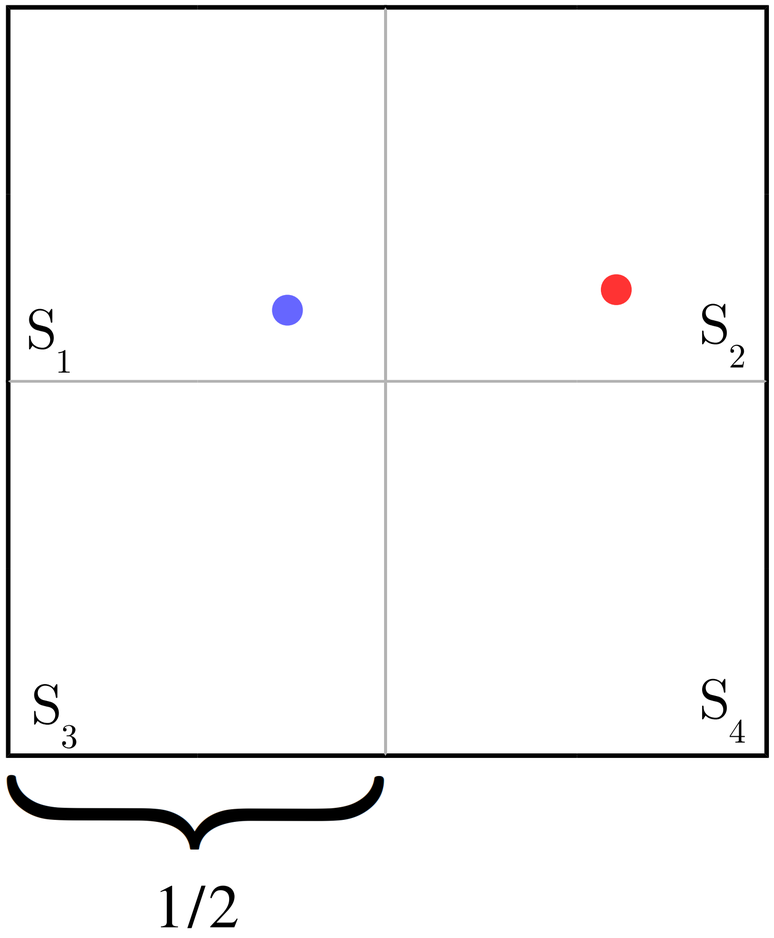
$$\mathcal{Q}_1 = \{S_1, S_2, S_3, S_4\}$$



$$\mu = \frac{1}{n}\sum_{i=1}^{n}\delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n}\delta_{y_i}$$

$$W_1(\mu,\nu) \leq \frac{\sqrt{d}}{2} + \dots$$

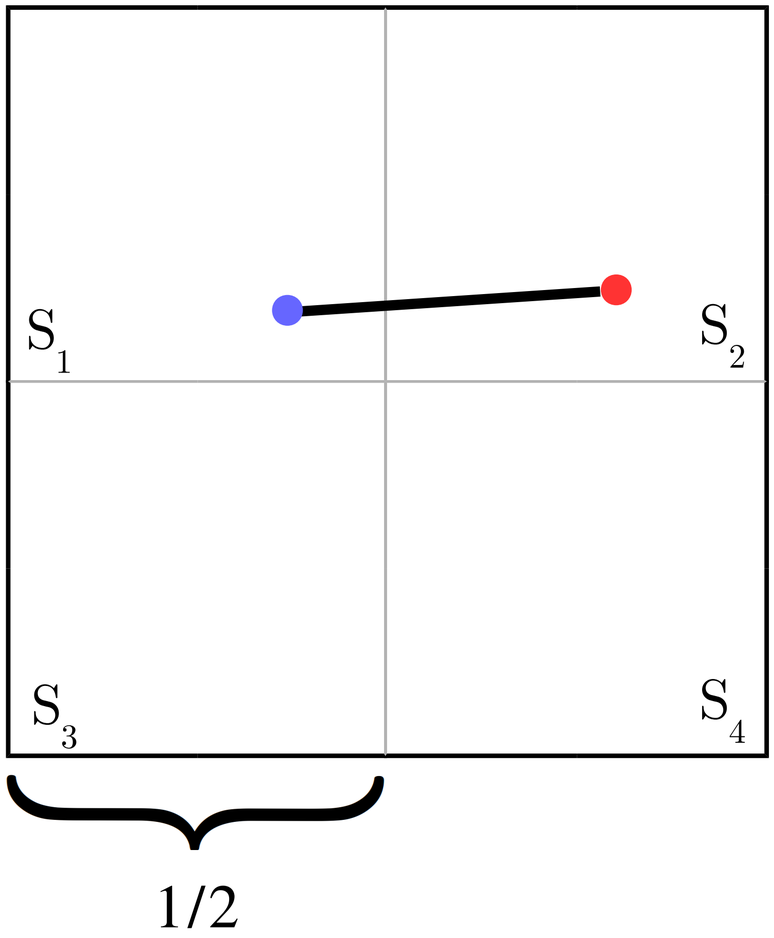
$$\mathcal{Q}_1 = \{S_1, S_2, S_3, S_4\}$$



$$\mu = \frac{1}{n}\sum_{i=1}^{n}\delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n}\delta_{y_i}$$

$$W_1(\mu,\nu) \leq \frac{\sqrt{d}}{2} + \dots$$

$$\mathcal{Q}_1 = \{S_1, S_2, S_3, S_4\}$$

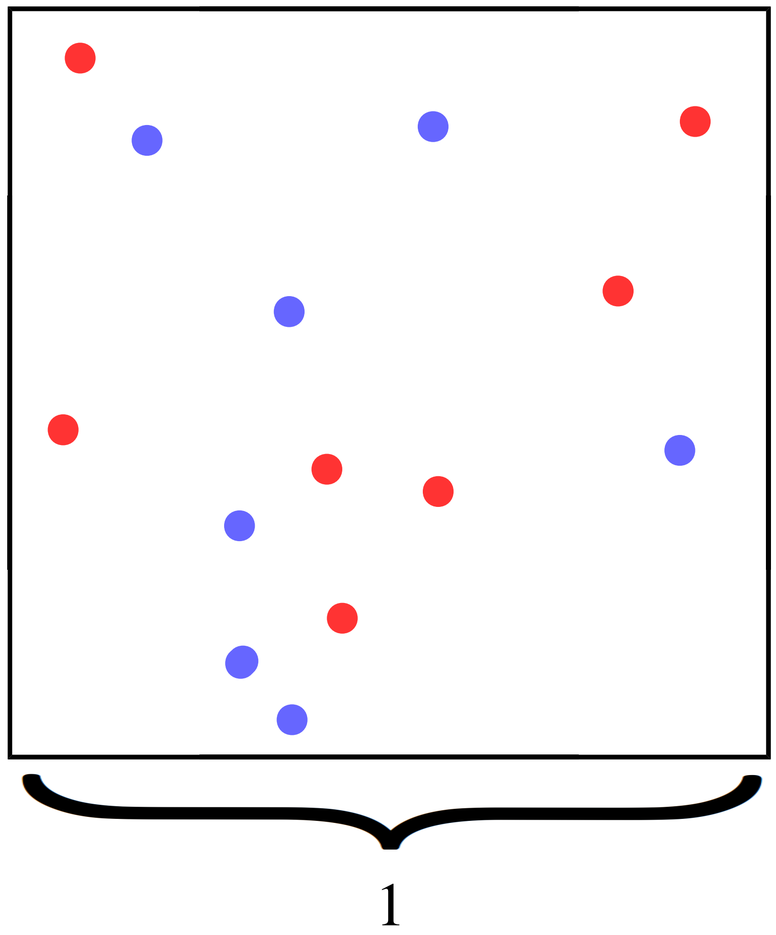


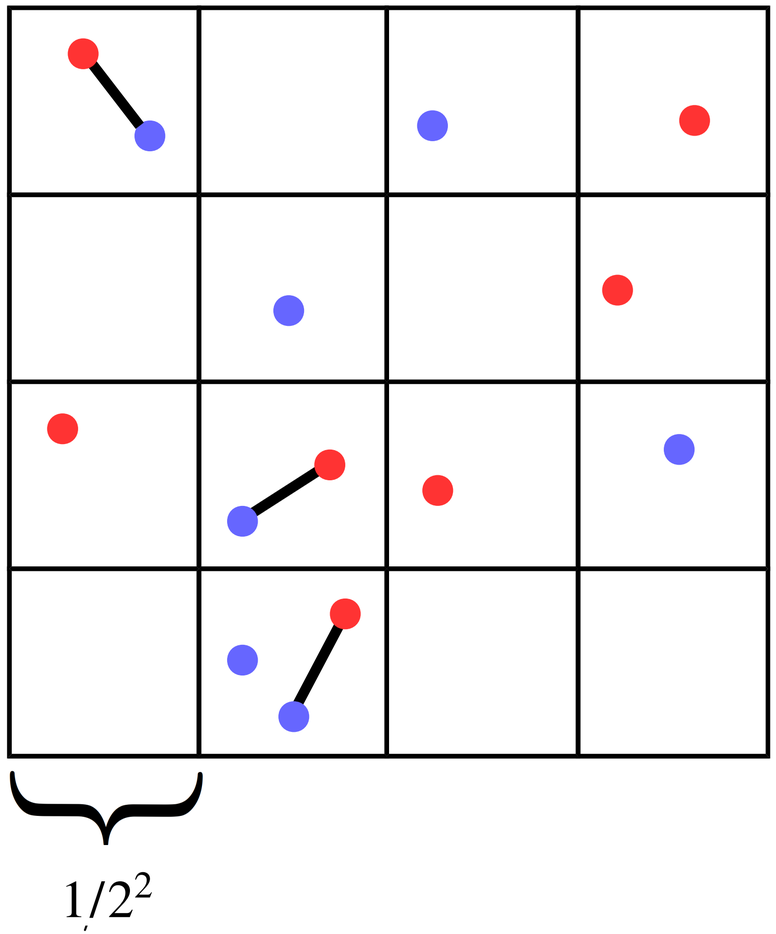
$$\mu = \frac{1}{n}\sum_{i=1}^{n}\delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n}\delta_{y_i}$$

$$W_1(\mu,\nu) \leq \frac{\sqrt{d}}{2} + \sqrt{d}\sum_{S\in\mathcal{Q}_j} |\,\mu(S) - \nu(S)\,|$$

$$\mathcal{Q}_1 = \{S_1, S_2, S_3, S_4\}$$

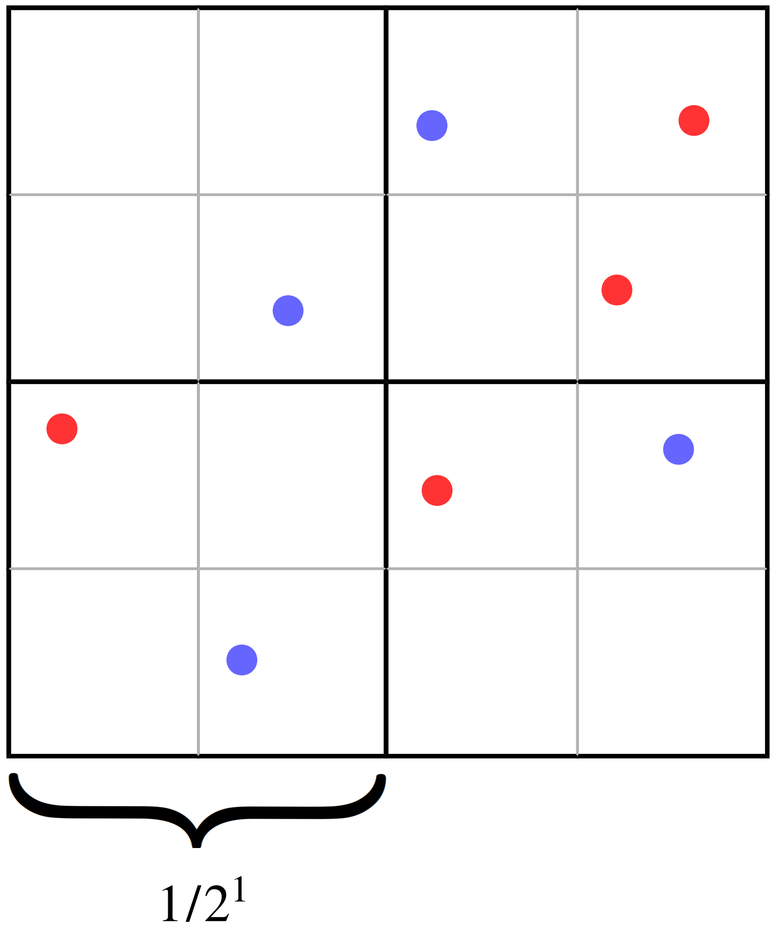
$$\mu = \frac{1}{n}\sum_{i=1}^{n} \delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n} \delta_{y_i}$$





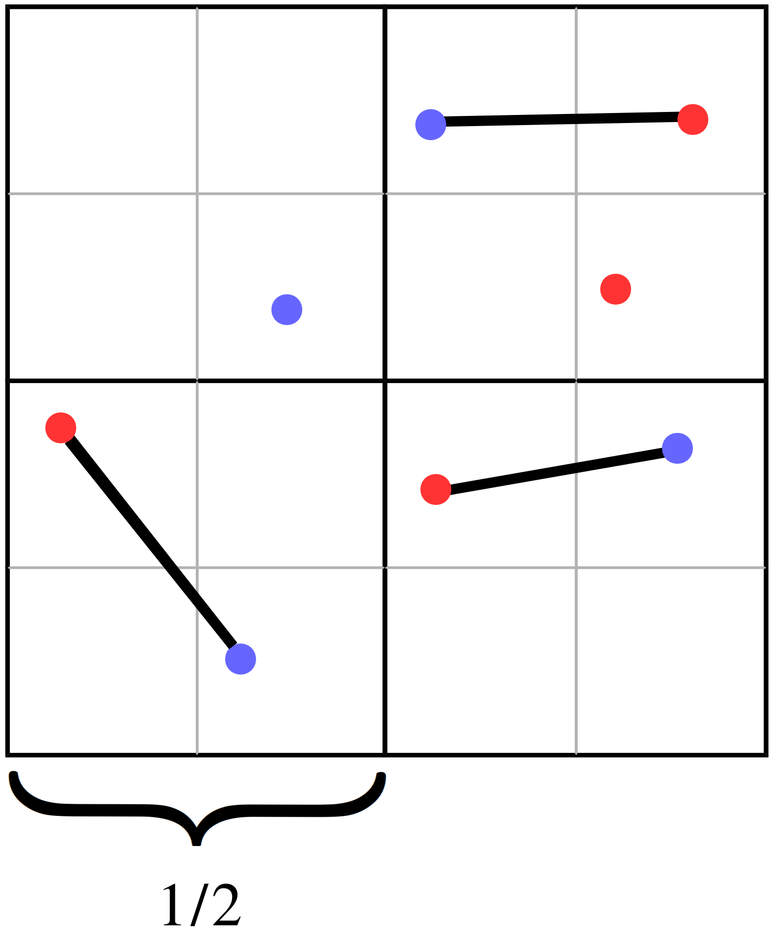
$$\mu = \frac{1}{n}\sum_{i=1}^{n} \delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n} \delta_{y_i}$$

$$W_1(\mu,\nu) \leq \frac{\sqrt{d}}{2^2} + \dots$$



$$\mu = \frac{1}{n}\sum_{i=1}^{n}\delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n}\delta_{y_i}$$

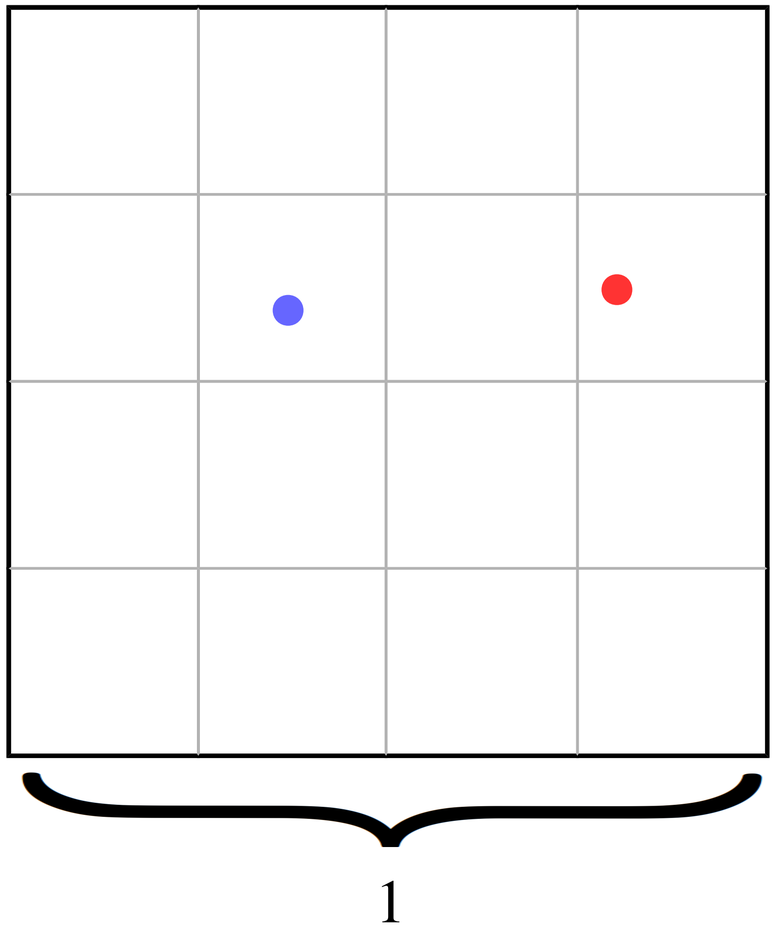
$$W_1(\mu,\nu) \leq \frac{\sqrt{d}}{2^2} + \dots$$



$$\mu = \frac{1}{n}\sum_{i=1}^{n}\delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n}\delta_{y_i}$$

$$W_1(\mu,\nu) \le \frac{\sqrt{d}}{2^2} + \frac{\sqrt{d}}{2^1}\sum_{S\in\mathcal{Q}_2} |\mu(S) - \nu(S)|$$

$$\mathcal{Q}_2 = \{\text{cubes of side length } 1/4\}$$

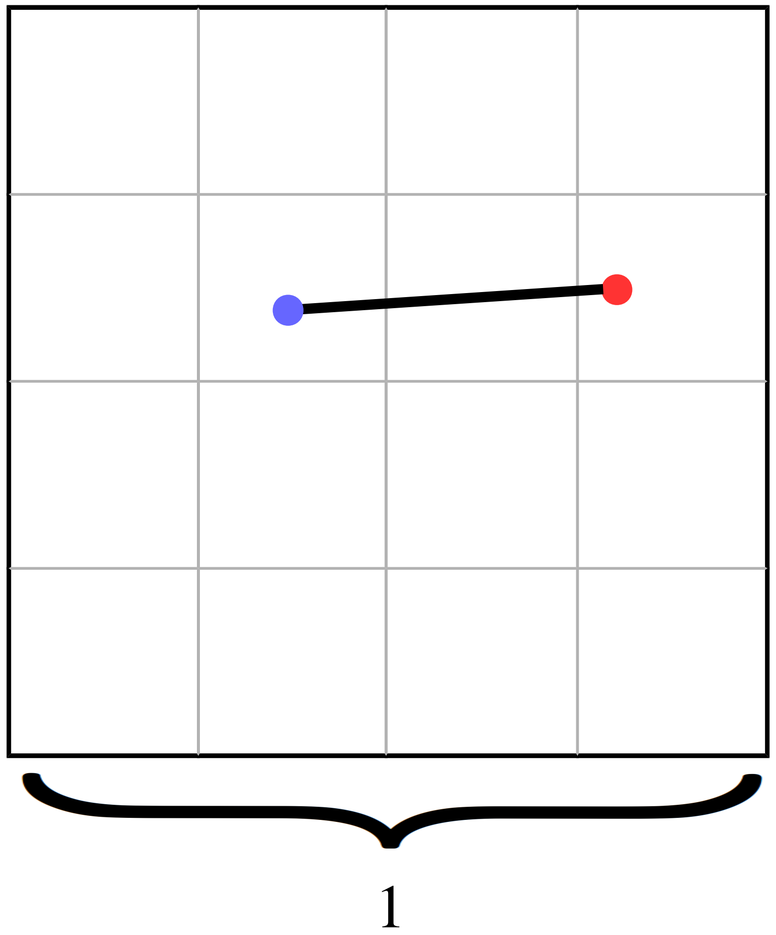


$$\mu = \frac{1}{n}\sum_{i=1}^{n}\delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n}\delta_{y_i}$$

$$W_1(\mu,\nu) \le \frac{\sqrt{d}}{2^2} + \frac{\sqrt{d}}{2^1}\sum_{S\in\mathcal{Q}_2} |\mu(S) - \nu(S)|$$

$$\mathcal{Q}_2 = \{\text{cubes of side length } 1/4\}$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} \delta_{x_i}, \quad \nu = \frac{1}{n}\sum_{i=1}^{n} \delta_{y_i}$$



$$W_1(\mu,\nu) \le \frac{\sqrt{d}}{2^2} + \frac{\sqrt{d}}{2^1}\sum_{S\in\mathcal{Q}_2} |\mu(S)-\nu(S)| + \sqrt{d}\sum_{S\in\mathcal{Q}_1} |\mu(S)-\nu(S)| \Bigg)$$

$\mathcal{Q}_2 = \{\text{cubes of side length } 1/4\}$

$\mathcal{Q}_1 = \{\text{cubes of side length } 1/2\}$