Math 164: Homework 1

Due Friday, April 3rd

Questions followed by * are to be turned in. Questions without * are extra practice. At least one extra practice questions will appear on each exam.

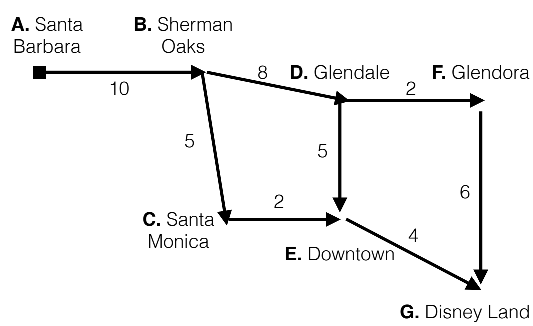
Question 1* (Section 1.4)

Suppose you work in operations at Apple and are tasked with planning production for some newly released products. Demand for these products is extremely high, and you must decide how to best allocate the company's limited raw materials and manpower in order to maximize monthly revenue. The table below indicates the amount of materials and labor required to assemble each new product. Suppose that 8000 units of materials and 2000 units of labor are available. Formulate a linear program to maximize revenue.

Product	Labor	Materials	Revenue
iPhone 6	6	12	\$199
iPhone 6 Plus	6	14	\$299
Apple Watch	10	12	\$349
Apple Watch Edition	10	120	\$2,999

Question 2* (Section 1.6)

Suppose you want to find the maximum number of cars per hour that can get from Santa Barbara to Disneyland. The traffic network is represented in the figure below. The numbers on each arrow represent the capacity of that road in thousands of cars per hour. Assume that all cars that enter an intersection also leave the intersection. Formulate a linear program to maximize the flow rate of cars through the network. (Hint: if x_{AB} is the flow rate of cars from Santa Barbara to Sherman Oaks, you should maximize x_{AB}. By the rule that all cars that enter an intersection must leave that intersection, all cars that leave Santa Barbara will eventually get to Disney Land.)



Question 3* (Friedburg, *Linear Algebra*, "Systems of Linear Equations")

For the following system of linear equations, find the coefficient matrix A. Recall that for any $m \times n$ matrix A, the *rank* of the matrix is the number of linearly independent rows, which turns out to always equal to the number of linearly independent columns. Determine the rank of the coefficient matrix A. Then find the dimension of and a basis for the solution set.

$$\begin{cases} x_1 + 2x_2 &= 0 \\ 2x_1 + 4x_2 &= 0 \end{cases}$$

Now find all solutions to the following system.

$$\begin{cases} x_1 + 2x_2 &= 5 \\ 2x_1 + 4x_2 &= 10 \end{cases}$$

Question 4* (Friedburg, *Linear Algebra*, “Systems of Linear Equations”)

For the following system of linear equations, find the coefficient matrix A and determine the rank of A. Then find the dimension of and a basis for the solution set.

$$\begin{cases} x_1 + x_2 - x_3 &= 0 \\ 4x_1 + x_2 - 3x_3 &= 0 \end{cases}$$

Now find all solutions to the following system.

$$\begin{cases} x_1 + x_2 - x_3 &= 1 \\ 4x_1 + x_2 - 3x_3 &= 3 \end{cases}$$

Question 5* (Section 1.3)

Suppose the following points lie on the graph of a quadratic function $(t, b(t))$,

$$(0, 1),\ (2, 7),\ (4, 37)\ .$$

Find the quadratic function and plot its graph.

Question 6 (Friedburg, *Linear Algebra*, “Systems of Linear Equations”)

Consider the system of linear equations

$$\begin{cases} x_1 + 4x_2 &= 4 \\ 2x_1 + 5x_2 &= 3\ . \end{cases}$$

What is the coefficient matrix A? Is A invertible? If so, what is A^{-1}? Use A^{-1} to solve the system.

Question 7* (Similar to Textbook Problem 2.2.1)

Consider the feasible region defined by the constraints

$$4 - x_1^2 - x_2^2 \geq 0\ , \sqrt{5} - x_1 - x_2 \geq 0\ , \text{ and } x_2 \geq 0\ .$$

For each of the following points, determine whether the point is feasible or infeasible, and (if it is feasible) whether it is interior to or on the boundary of each of the constraints: $x_a = (1, 1)^T$, $x_b = (2, 0)^T$, $x_c = (-2, 0)^T$, $x_d = (-1/2, 0)^T$, and $x_e = (1/\sqrt{5}, 1/\sqrt{5})^T$.

Question 8* (Similar to Textbook Problem 2.2.3)

Consider the problem

$$\begin{aligned} \text{minimize } f(x) &= x_1\ , \\ \text{subject to } x_1^2 + x_2^2 &\leq 9 \\ x_1^2 &\geq 1\ . \end{aligned}$$

Graph the feasible set. Use the graph to find all local minimizers for the problem, and determine which of those are also global minimizers.

Question 9 (Textbook Problem 2.2.4)

Consider the problem

$$\begin{aligned} \text{minimize } & f(x) = x_1 \ , \\ \text{subject to } & (x_1-1)^2 + x_2^2 = 1 \\ & (x_1+1)^2 + x_2^2 = 1 \ . \end{aligned}$$

Graph the feasible set. Are there local minimizers? Are there global minimizers?

Question 10 (Textbook Problem 2.2.5)

Give an example of a function f and a feasible set S so that f has no global minimizer and no global maximizer on S.