Math 164: Homework 3
Due Friday April 17th

Questions followed by * are to be turned in. Questions without * are extra practice. At least one extra practice question will appear on each exam.

**Question 1 (Similar to Textbook Problem 3.1.2)**

Consider the set defined by the constraints $x_2 - x_1 = 0$, $x_1 \leq 1$, and $x_2 \leq 1$. At each of the following points determine the set of feasible directions: $x_a = (0, 0)^T$, $x_b = (1, 1)^T$, $x_c = (0.5, 0.5)^T$.

**Question 2* (Similar to Textbook Problem 4.1.1)**

Consider the problem

\[
\begin{align*}
\text{minimize} & \quad f(x), \\
\text{subject to} & \quad x_1 + 2x_2 + 4x_3 = 8, \\
& \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.
\end{align*}
\]

(a) Find the set of all feasible directions at points $x_a = (0, 0, 2)^T$, $x_b = (2, 1, 1)^T$, $x_c = (6, 1, 0)^T$.

(b) Using part (a), verify that $p = (-4, 0, 1)^T$ is a feasible direction for $x_c = (6, 1, 0)^T$. Then find an upper bound on the step length $\alpha$ so that $x_c + \alpha p$ is a feasible point.

**Question 3* (Similar to Textbook Problem 4.1.1)**

Consider the linear program

\[
\begin{align*}
\text{minimize} & \quad f(x), \\
\text{subject to} & \quad x_1 - x_2 \leq 1, \\
& \quad x_1 + x_2 \leq 1, \\
& \quad x_1 \geq 0.
\end{align*}
\]

For the following choices of $f(x)$, solve the linear program graphically, i.e. find a global minimizer for $f(x)$ or show that none exists: (a) $f(x) = -x_1$, (b) $f(x) = x_2$, (c) $f(x) = -x_1 - x_2$.

Do any of the functions have more than one global minimizer?

**Question 4 (Similar to Textbook Problem 4.1.1)**

Fix $a > 0$. Solve the linear program graphically, i.e. find a global minimizer for $f(x)$ or show none exists.

\[
\begin{align*}
\text{minimize} & \quad f(x) = x_1 - 2x_2, \\
\text{subject to} & \quad x_1 + x_2 \leq a, \\
& \quad x_1 \geq 0, \\
& \quad x_2 \geq 0.
\end{align*}
\]
Question 5 (Similar to Textbook Problem 4.1.1)

Solve the linear program graphically, i.e. find a global minimizer for \( f(x) \) or show that none exists.

\[
\begin{align*}
\text{minimize} & \quad -x_1 + 2x_2, \\
\text{subject to} & \quad 5x_1 + 2x_2 \geq 10, \\
& \quad 2x_1 + 3x_2 \leq 40, \\
& \quad x_1 \leq 15, \\
& \quad x_2 \leq 15.
\end{align*}
\]

Question 6* (Similar to Textbook Problem 4.1.1)

Solve the linear program graphically, i.e. find a global minimizer for \( f(x) \) or show that none exists.

\[
\begin{align*}
\text{minimize} & \quad -x_1 - x_2, \\
\text{subject to} & \quad x_2 - x_1 \geq 0, \\
& \quad x_2 - 2x_1 \geq 2, \\
& \quad x_1 \geq 0, \\
& \quad x_2 \geq 0.
\end{align*}
\]

Question 7* (Similar to Textbook Problem 4.1.1)

Solve the linear program graphically, i.e. find a global minimizer for \( f(x) \) or show that none exists.

\[
\begin{align*}
\text{minimize} & \quad \pi x_1 + ex_2, \\
\text{subject to} & \quad x_1 + x_2 \leq 6, \\
& \quad x_2 - x_1 \geq 3, \\
& \quad 2x_1 - x_2 \geq 2, \\
& \quad x_1 \geq 0, \\
& \quad x_2 \geq 0.
\end{align*}
\]

no question 8?

Question 9* (Similar to Textbook Problem 4.2.2)

Convert the following linear program to standard form:

\[
\begin{align*}
\text{minimize} & \quad z = x_1 - 5x_2 - 7x_3, \\
\text{subject to} & \quad 3x_1 - x_2 + 9x_3 \geq 7, \\
& \quad 5x_1 + 0x_2 - 3x_3 = 1, \\
& \quad 7x_1 + 5x_2 + 5x_3 \leq 9, \\
& \quad x_1 \geq -2, \\
& \quad x_2,x_3 \text{ free}.
\end{align*}
\]

Question 10 (Similar to Textbook Problem 4.2.3)

Convert the linear program in Question 5 to standard form.