

Directed Algebraic Topology

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What Is Directed Algebraic Topology?

- Topological spaces do not have a notion of direction
- But natural objects like digraphs or spacetimes do...

- Directed Algebraic Topology is the study of spaces endowed with a notion of direction (or a set of allowed paths)

Why Directed Algebraic Topology?

- Study state spaces of concurrent programs
- Study conal manifolds (i.e. a manifold M with choice of a positive cone $C_x \subset T_x M$ for all $x \in M$)
- Many other possible applications...

Definition [Grandis, 2001]

A *dspace* (X, dX) is a topological space X together with dX , a set of paths $[0, 1] \rightarrow X$ which is closed under constant paths, concatenation, and monotone increasing reparametrization.

A *dmap* $f : (X, dX) \rightarrow (Y, dY)$ is a continuous map $f : X \rightarrow Y$ such that $f(dX) \subset dY$.

Definition

A *dihomotopy* $H : X \times [0, 1] \rightarrow Y$ between $f, g : (X, dX) \rightarrow (Y, dY)$ is a homotopy between f and g such that $H(\cdot, t)$ is a dmap for $0 \leq t \leq 1$.

-  [Marco Grandis \(2001\)](#)
Directed homotopy theory, I. The fundamental category
[ArXiv:math.AT/0111048v2](#)
-  [Sanjeevi Krishnan \(2012\)](#)
Cubical Approximation For Directed Spaces
[ArXiv:1012.0509v2](#)