Math 8 Extra Problems

Date: Summer 2021

Name: Euler

1. Prove that there do not exist positive integers a, b, c and n such that

$$a^2 + b^2 + c^2 = 2^n abc.$$

2. Find all functions f which satisfy the three conditions

   (a) f(x,x)=x

   (b) f(x,y)=f(y,x)

   (c) f(x,y)=f(x,x+y)

   assuming that the variables and the values of f are positive integers.

3. If $\gcd(a,b) = 1$, prove that

   (a) $\gcd(a-b, a+b) \leq 2$,

4. Prove that if for integers a and b we have $7|(a^2+b^2)$, then $7|a$ and $7|b$.

5. Prove that for positive integer n we have $n^2|(n+1)^n - 1$

6. Prove that there is no polynomial with integer coefficients $p(x)$ with the property $p(7) = 5$ and $p(15) = 9$.

7. prove or disprove: If a and b are odd integers, then $4|(a-b)$ or $4|(a+b)$.

8. Prove that if m and n are natural numbers that

$$3^m + 3^n + 1$$

   cannot be a perfect square.

9. Prove that every integer greater than 6 can be represented as a sum of two integers greater than 1 which are relatively prime.