Math 8 Week 1
June 25, 2021

Quiz 1: Sets and Proofs Solutions

1. (3 points) Let $a \in \mathbb{Z}$. State the converse, inverse, and contrapostive of the following statement:

   If a is even, then $a+1$ is odd.

   Note: Make sure you label which is the converse, which is the inverse, etc.
   Converse: If $a+1$ is odd, then a is even.

   Inverse: If a is not even, then $a+1$ is not odd. OR If a is odd, then $a+1$ is even

   Contrapositive: If $a+1$ is not odd, then a is not even. OR If $a+1$ is even, then a is odd.

2. (3 points) Let A and B be sets. Prove that if $A \cap B = A$, then $A \subseteq B$.
   1. DIRECT PROOF:

   Proof. Let $x \in A$. Then, $x \in A \cap B$ since $A = A \cap B$. So, by definition $x \in B$. Hence $A \subseteq B$. □

   2. PROOF BY CONTRAPOSITIVE:

   Proof. Let A and B be sets such that $A \not\subseteq B$.Then there exist an x in A but not in B by the negation of the definition of subset. So $x \notin A \cap B$. But $x \in A$. Hence, $A \neq A \cap B$. □

3. (4 points) Let A and B be sets. Prove that if $A \subseteq B$, then $A \cap B = A$.
   1. DIRECT PROOF:

   Proof. Let A and B be sets such that $A \subseteq B$. Note that if $A = \varnothing$, then $A \cap B = \varnothing$ and the statement is true. Further, if $B = \varnothing$ then $A = \varnothing$ since $A \subset B$ and $A \cap B = \varnothing = A$. Therefore, we assume $A, B \neq \varnothing$. We claim $A \cap B \subseteq A$.

   Proof of claim: Let $x \in A \cap B$. Then, by definition of intersection, $x \in A$. So, by definition of subset, $A \subseteq A \cap B$.

   Now, we claim $A \subseteq A \cap B$.

   Proof of claim: Let $x \in A$. Since $A \subseteq B$, by definition of subset, $x \in B$. So, $x \in A$ and $x \in B$. By definition of intersection, $x \in A \cap B$. Therefore, $A \subseteq A \cap B$.

   Since $A \cap B \subseteq A$ and $A \subseteq A \cap B$, we conclude that $A \cap B = A$. □

   2. PROOF BY CONTRAPOSITIVE:

Proof. Let A and B be sets such that $A \cap B \neq A$. Then, by definition there either exists $x \in A \cap B$ such that $x \notin A$, or there exists $x \in A$ such that $x \notin A \cap B$. Note that the first case cannot be true since by definition $A \cap B \subseteq A$. Therefore, suppose that there exists $x \in A$ such that $x \notin A \cap B$. In particular, this implies that $x \notin B$. So, $A \nsubseteq B$. $\square$

Note: Putting these two proofs together, you've proven the following statement: Let A and B be sets. $A \cap B = A$ if and only if $A \subset B$.