WORKSHEET 13

Date: 11/22/2021

Name:

Partitions and Functions

DEFINITION 1. Let n be a positive integer, and let S be a set. A **partition** of S is a collection of subsets $S_1, S_2, \ldots, S_k$ such that each element of S lies in exactly one of these subsets. In other words,

$$S = S_1 \cup S_2 \cup \ldots \cup S_k \text{ and } S_i \cap S_j = \phi \text{for any } i \neq j.$$

THEOREM 1. *Let S be a set and $\sim$ be an equivalence relation on S. Then, the equivalence classes of $\sim$ form a partition of S.*

REMARK 2. Note that the converse is also true. Let $P = \{A_\alpha : \alpha \in I\}$ be a partition of a nonempty set A. Then there exists an equivalence relation R defined on A such that $P = \{[a] : a \in A\}$ is the set of equivalence classes determined by R.

1. A relation R is defined on $\mathbb{Z}$ by xRy if $11x - 5y$ is even. Then R is an equivalence relation

   (a) Show R is an equivalence relation. Solution: You can show this.

   (b) Determine the equivalence class of the relation R.

2. Let $A = \mathbb{Z}$. Find four different partitions of the integers.

DEFINITION 2. Let X and Y be sets. A **function** f from X to Y is a *rule* that assigns to each element $x \in X$ a single element of Y, denoted by $f(x)$ or $y = f(x)$.

DEFINITION 3. If $f : X \to Y$ is a function, then

- The set X is called the **domain** of f;
- The **image** of f is the set of all elements of Y that are equal to $f(x)$ for some $x \in X$. We write $f(X)$ for the image of f, and it is specifically,

  $$f(X) = \{f(x) | x \in X\}.$$

  Let X be any set. We define a function $i_x : X \to X$, called the *identity* of X by

  $$i_x(x) = x$$

DEFINITION 4. Let $f : X \to Y$ be a function. We say that

- f is **onto** or **surjective** if the image $f(X) = Y$. i.e, for every $y \in Y$, there exists an $x \in X$ such that $f(x) = y$.
- f is **one-to-one** or **injective** if for $a, b \in X$

  $$\text{if } f(a) = f(b) \text{ then } a = b.$$

- f is a **bijection** if f is both surjective and injective.

1. Prove $f : \mathbb{R} \to \mathbb{R}$ is a bijection when
$$f(x) = \frac{3x+5}{7}.$$

2. Let X be a set and $f : X \to X$ be a function such that $f(f(x)) = x$ for all x. Then f is a bijection.