WORKSHEET 10

Date: 11/08/2021

Name:

F.T.A and Congruence of Integers

THEOREM 1 (Fundamental Theorem of Arithmetic)**.** *Every integer greater than 1 can be factored uniquely into a product of primes, up to ordering.*

PROPOSITION 2. *Let p be a prime. Prove $\sqrt{p}$ is irrational*

You should be able to prove the following:

PROPOSITION 3. *Let p be a prime. Prove $\sqrt[n]{p}$ is irrational when $n \geq 2$.*

PROPOSITION 4. *Prove that the only prime of the form $n^3 - 1$ is 7.*
Note that $n^3 - 1 = (n-1)(n^2+n+1)$.

PROPOSITION 5. *Prove there is one and only one natural number n such that* $2^8 + 2^{11} + 2^n$ *is a perfect square.*

DEFINITION 1 (Congruence of Integers)**.** Let m be a positive integer. For $a, b \in \mathbb{Z}$, if m divides $b - a$, we say that a is **congruence to** b **modulo** m, written as

$$a \equiv b \pmod{m}$$

1. let us endeavor to show that 41 divides $2^{20} - 1$.

2. For another example in the same spirit, suppose we are asked to find the remainder obtained upon dividing the sum

$$\sum_{n=1}^{100} n!$$

by 12. Without the aid of concurrences this would be an awesome calculation.

3.

THEOREM 6 (Cancellation Law for Congruence). *If* $ca \equiv cb \pmod{n}$, *then* $a \equiv b \pmod{n/d}$, *where* $d = gcd(c,n)$.

4. Show that if n is an odd positive integer, then 11 divides $5^n + 6^n$.