WORKSHEET 1

Date: 09/27/2022

Name:

Definitions

DEFINITION 1 (Set)**.**

DEFINITION 2 (union)**.**

DEFINITION 3 (intersection)**.**

DEFINITION 4 (set difference)**.**

DEFINITION 5 (subset)**.**

DEFINITION 6 (equality of sets)**.**

DEFINITION 7 (empty set)**.**

DEFINITION 8 (Cartesian product)**.**

DEFINITION 9 (cardinality)**.**

Example 1.6 Be sure you understand why the following statements are true. Each illustrates an aspect of set theory that you've learned so far.

1. $1 \in \{1,\{1\}\}$ ..... 1 is the first element listed in $\{1,\{1\}\}$
2. $1 \notin \{1,\{1\}\}$ ..... because 1 is not a set
3. $\{1\} \in \{1,\{1\}\}$ ..... $\{1\}$ is the second element listed in $\{1,\{1\}\}$
4. $\{1\} \subseteq \{1,\{1\}\}$ ..... make subset $\{1\}$ by selecting 1 from $\{1,\{1\}\}$
5. $\{\{1\}\} \notin \{1,\{1\}\}$ ..... because $\{1,\{1\}\}$ contains only 1 and $\{1\}$, and not $\{\{1\}\}$
6. $\{\{1\}\} \subseteq \{1,\{1\}\}$ ..... make subset $\{\{1\}\}$ by selecting $\{1\}$ from $\{1,\{1\}\}$
7. $\mathbb{N} \notin \mathbb{N}$ ..... $\mathbb{N}$ is a set (not a number) and $\mathbb{N}$ contains only numbers
8. $\mathbb{N} \subseteq \mathbb{N}$ ..... because $X \subseteq X$ for every set X
9. $\emptyset \notin \mathbb{N}$ ..... because the set $\mathbb{N}$ contains only numbers and no sets
10. $\emptyset \subseteq \mathbb{N}$ ..... because $\emptyset$ is a subset of every set
11. $\mathbb{N} \in \{\mathbb{N}\}$ ..... because $\{\mathbb{N}\}$ has just one element, the set $\mathbb{N}$
12. $\mathbb{N} \not\subseteq \{\mathbb{N}\}$ ..... because, for instance, $1 \in \mathbb{N}$ but $1 \notin \{\mathbb{N}\}$
13. $\emptyset \notin \{\mathbb{N}\}$ ..... note that the only element of $\{\mathbb{N}\}$ is $\mathbb{N}$, and $\mathbb{N} \neq \emptyset$
14. $\emptyset \subseteq \{\mathbb{N}\}$ ..... because $\emptyset$ is a subset of every set
15. $\emptyset \in \{\emptyset,\mathbb{N}\}$ ..... $\emptyset$ is the first element listed in $\{\emptyset,\mathbb{N}\}$
16. $\emptyset \subseteq \{\emptyset,\mathbb{N}\}$ ..... because $\emptyset$ is a subset of every set
17. $\{\mathbb{N}\} \subseteq \{\emptyset,\mathbb{N}\}$ ..... make subset $\{\mathbb{N}\}$ by selecting $\mathbb{N}$ from $\{\emptyset,\mathbb{N}\}$
18. $\{\mathbb{N}\} \not\subseteq \{\emptyset,\{\mathbb{N}\}\}$ ..... because $\mathbb{N} \notin \{\emptyset,\{\mathbb{N}\}\}$
19. $\{\mathbb{N}\} \in \{\emptyset,\{\mathbb{N}\}\}$ ..... $\{\mathbb{N}\}$ is the second element listed in $\{\emptyset,\{\mathbb{N}\}\}$
20. $\{(1,2),(2,2),(7,1)\} \subseteq \mathbb{N} \times \mathbb{N}$ ..... each of $(1,2)$, $(2,2)$, $(7,1)$ is in $\mathbb{N} \times \mathbb{N}$

Practice Problems

1. Let $A = \{a, b, c\}, B = \{c, h\}, C = \{a, d\}$. Compute the "operations" below.

   (a) $A \cup B$

   (b) $A \cap B$

   (c) $A \setminus B$

   (d) $A \cap B \cap C$

   (e) $A \setminus (B \cup C)$

2. For each of the following sets, list the elements. For infinite sets, list five of the elements.

   (a) $\{x^3 : x \in \mathbb{Z}\}$

   (b) $\{3x : |2x| < 3, x \in \mathbb{Z}\}$

3. Write each of the following in set-builder notation

   (a) $\{1, 3, 5, 7, \ldots\}$

   (b) $\{3, 6, 9, 12, \ldots\}$

   (c) $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\}$

4. Find the cardinality of each of the following sets.

   (a) $|\{0, 1, i, \pi, e\}|$

(b) $|\{\phi, \{\phi\}\}|$

5. Let $A = \{a, b, c\}, B = \{0, 1\}$. For each of the following sets, list the elements.

   (a) $A \times B$

   (b) $B \times A$

   (c) $A \times A$

   (d) A^3

6. List all of the subsets of the following sets.

   (a) $\{a, b\}$

   (b) $\{x, y, z\}$