Math 8 Week 6
July 30, 2021

Quiz 5

1. (4 points) Use the Euclidean algorithm to find $hcf(74, 383)$. Then find $s, t \in \mathbb{Z}$ such that $74s + 383t = d$.

   Note that

   $$\begin{aligned} 383 &= 5 \cdot 74 + 13 \\ 74 &= 5 \cdot 13 + 9 \\ 13 &= 1 \cdot 9 + 4 \\ 9 &= 2 \cdot 4 + 1 \\ 4 &= 4 \cdot 1 + 0 \end{aligned}$$

   Therefore, by the Euclidean Algorithm, we know that $hcf(74, 383) = 1$. Now, the previous calculations imply that:

   $$\begin{aligned} 383 - 5 \cdot 74 &= 13 \\ 74 - 5 \cdot 13 &= 9 \\ 13 - 1 \cdot 9 &= 4 \\ 9 - 2 \cdot 4 &= 1 \end{aligned}$$

   Therefore,

   $$\begin{aligned} 1 &= 9 - 2 \cdot 4 \\ &= 9 - 2(13 - 9) = 3 \cdot 9 - 2 \cdot 13 \\ &= 3 \cdot (74 - 5 \cdot 13) - 2 \cdot 13 = 3 \cdot 74 - 17 \cdot 13 \\ &= 3 \cdot 74 - 17 \cdot (383 - 5 \cdot 74) \\ &= -17 \cdot 383 + 88 \cdot 74 \end{aligned}$$

   Therefore, $s = 88$ and $t = -17$.

2. (4 points) Let m, n be coprime integers, and suppose a is an integer which is divisible by both m and n. Prove that mn divides a.

   Proof. Let m, n be coprime integers, and suppose a is an integer which is divisible by both m and n. Then, by definition $hcf(m, n) = 1$. So, by Proposition 10.3, there exists $s, t \in \mathbb{Z}$ such that

   $$ms + nt = 1$$

   Therefore,

   $$ams + ant = a$$

   Since m and n both divide a, we know that $mx = a$ and $ny = a$ for some $x, y \in \mathbb{Z}$. Substituting these into the equation above we have,

   $$nyms + mxnt = a$$

   So, $nm(ys + xt) = a$ and since $ys + xt \in \mathbb{Z}$ we conclude that mn divides a. □

3. (2 points) Is the conclusion of 2 necessarily true if m and n are not coprime? In other words, prove or disprove the following statement: Let a, m, and n be integers such that $hcf(m, n) > 1$. If $m|a$ and $n|a$, then $mn|a$.

   The conclusion of 2 is not necessarily true if m and n are not prime. For example, let $m = 6$, $n = 3$ and $a = 12$. Then, m divides a and n divides a. But, $mn = 18$ does not divide $a = 12$.