WORKSHEET 4

Date: 04/06/2022

Name:

PROOFS AND COUNTER-EXAMPLES

As the title suggests, we will go over proofs and counter-examples of statements in section today. I found a web-page which goes over basic proof techniques. I suggest you click the following link and explore the page. I would love to go over this in section, but unfortunately we don't have enough time. Click me please.

Perfect Proof Practice

Break into groups and construct a proof for the following questions. You group will volunteer one represented, Squid Game style, to attempt a proof on the chalkboard.

1. If $C \subseteq A$, $D \subseteq B$, and A and B are disjoint, then C and D are disjoint.
   [Recall: two sets are disjoint if there intersection is empty i.e $A \cap B = \emptyset$]

2. If $A \cup B \subseteq C \cup D$, $A \cap B = \varnothing$, and $C \subseteq A$, then $B \subseteq D$.

3. Let x and y be integers. If x and y are odd integers, then there does not exist an integer z such that $x^2 + y^2 = z^2$.
   [Hint: a proof by contradiction should follow easily.]

4. Prove that $\{x \in \mathbb{Z} : 18|x\} \subseteq \{x \in \mathbb{Z} : 6|x\}$.
   [Notation: we say $18|x$ if and only if there exists an integer z such that $x = 18z$. In general if $a \neq 0, b$ are integers then we say $a|b$ if and only if there exists an integer c such that $b = ac$.]

CONCOCTING CONCISE COUNTER-EXAMPLES

How do we show a conditional statement is false?
Recall the truth table for the conditional statement $P \Rightarrow Q$. The only way this statement is false is when P is true and Q is false. Our job is to come up with a cleaver counter example to satisfy the condition we want. Lets do some examples to help solidify this idea.

Prove or disprove the following statements below.

(a) For every rational number q, there is a rational number r such that $qr = 1$.

(b) If q is rational and x is irrational, then qx is irrational.

(c) Assume $p_1, p_2, \ldots, p_n$ are the first n primes , then $(p_1 p_2 \ldots p_n) - 1$ is prime.

5.

THEOREM 1. *e is irrational.*