WORKSHEET 2^2

Date: 06/27/2022

Name:

1. Prove the following important statement:

$$\forall \varepsilon > 0, |x - y| \leq \varepsilon \Leftrightarrow x = y.$$

2. In this problem, we denote (a,b) as the interval $\{x \in \mathbb{R} | a < x < b\}$. Let A be a subset of $\mathbb{R}$. We say that A is *open*, if

$$(O) \quad \text{for any } a \in A, \text{ there is } \varepsilon > 0, \text{ such that } (a - \varepsilon, a + \varepsilon) \subset A.$$

(a) Write down the negation of (O) as a complete sentence.

(b) Determine whether the empty set ϕ is open.

(c) Show that $(0,1)$ is open.

(d) Show that $[0,1]$ is not open, where $[0,1] = \{x \in \mathbb{R} | 0 \leq x \leq 1\}$.

3. Let S be a subset of $\mathbb{R}_+$. Suppose that S satisfies the property below:

$$\forall \varepsilon > 0, \exists s \in S \ni s \le \varepsilon.$$

Give three examples of S with this property (you don't need to prove anything).