1. Compute the following integrals:

   (a) $\int \left(e^{3x} - 3x^3\right) dx$

   (b) $\int \left(x^2 + 2x + 1\right) dx$

   (c) $\int \left(e^{-x} - e^x + \frac{1}{x}\right) dx$

2. Let $f(x) = x^3 - 3x$. (You might need a calculator for this problem)

   (a) Compute $\int_{-1}^{0} f(x)\ dx$

   (b) Compute a right Riemann approximation of $\int_{-1}^{0} f(x)\ dx$ with 6 rectangles.

   Recall that the right Riemann sum is

   $$\sum_{n=1}^{6} (\Delta x f(a + n\Delta x))$$

   where $\Delta x = \frac{b-a}{k}$.
   Lets be smart and denote $x_n = a + n\Delta x$. Then we have the following
   $f(x) = x^3 - 3x, a = -1, b = 0$ and $k = 6$.
   Thus, we get $\Delta x = \frac{1}{6}$ and
   $f(x_1) = f(\frac{-5}{6}) = \frac{415}{216}$
   $f(x_2) = f(\frac{-2}{3}) = \frac{46}{27}$
   $f(x_3) = f(\frac{-1}{2}) = \frac{11}{8}$
   $f(x_4) = f(\frac{-1}{3}) = \frac{26}{27}$
   $f(x_5) = f(\frac{-1}{6}) = \frac{107}{216}$
   $f(x_6) = f(0) = 0$
   Finally, we sum all these up and multiply by $\frac{1}{6}$. The answer is now

   $$\frac{1}{6}\left(\frac{415}{216} + \frac{46}{27} + \frac{11}{8} + \frac{26}{27} + \frac{107}{216}\right) = 1.076$$

   (c) Compute a left Riemann approximation of $\int_{-1}^{0} f(x)\ dx$ with 6 rectangles

   (d) Compute a midpoint Riemann approximation of $\int_{-1}^{0} f(x)\ dx$ with 6 rectangles

Cont.

3. Find the following averages:

   (a) 13 between $x = -6,249,837$ and $x = 123,456,789$

   (b) x^2 between $x = 0$ and $x = 1$.

   (c) x^2 between $x = -1$ and $x = 1$.

   (d) e^{3x} between $x = -t$ and $x = t$.

The End.