MATH 122A EXAM REVIEW

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Name: Hogarth

I want to make something very clear, these questions are my bias opinion of what questions I would study. The emphasis is on the I.

1. Find all the roots of $z^3 = \bar{z}$.
2. Find the roots of the following equation: $z^3 = (-1+i)$
3. Express $\left(\frac{-\sqrt{3}}{2} + \frac{i}{2}\right)^{603}$ in the form of $a+bi$.
4. For each of the following, identify the largest open disk on which the series converges. Justify your answer.
   (a)
   $$\sum_{n=0}^{\infty} [1+(-1)^n]^n z^n$$
   (b)
   $$\sum_{n=0}^{\infty} (3z+6)^n$$
5. In the following cases the boundary of S (The way the book defines it) can be described as the image of a path. Draw a picture of S and specify a function γ giving such a path.
   (i) $S = \{z \in \mathbb{C} : |z| \leq 1, imz \geq 0\}$
   (ii) $S = \{z \in \mathbb{C} : 1 \leq |z| \leq 2, imz \geq 0\}$
   (iii) $S = \{z \in \mathbb{C} : 0 \leq re\, z \leq 1, 0 \leq imz \leq 1\}$
   (iv) $S = \{z \in \mathbb{C} : 1 \leq |z| \leq 2, 0 \leq im\, z \leq re\, z\}$
6. Determine if the following sets are open or closed. Find the boundary of each set as well.
   (a) $A = \{z = x+iy : x \leq 1, y \leq 1\}$
   (b) $B = \{z = x+iy : x < 1, y < 1\}$
   (c) $C = \{z \in \mathbb{C} : |z| \leq 1, Im(z) > 0\}$
   (d) $D = \{z = x+iy : y = 0\}$
7. Determine whether the following sequences converge, and find the limits of those that converge.
   (i) $((1+i)^n)$
   (ii) $((1+i)^n/n!)$
8. What does $\sum_{n=0}^{\infty}(n+1)(\frac{1}{2})^n$ converge to? [Hint: how do we get this from $f(z) = 1/(1-z)$?]
9. Can a power series $\sum a_n(z-2)^n$ converge at $z = 2+2i$ but diverge at $z = 3$?

10. A holomorphic function f defined on a connected open set G must be constant on G if:

   (a) $f'(z) = 0$ for all $z \in G$
   (b) f takes only real values on G.
   (c) $\overline{f(z)}$ is a holomorphic function.
   (d) $|f|$ is constant on G.
   (e) at every point in G $f = 0$ or $f' = 0$. [Hint: consider f^2]

11. Suppose that f is analytic on the unit disc and that $Ref(z) = 3$ for all z in the unit disk. Then f is constant on the unit disk.