MATH 122A EXAM REVIEW

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I want to make something very clear, these questions are my bias opinion of what questions I would study. The emphasis is on the I.

1. Prove or disprove that
$$\int_0^{2\pi} e^{e^{i\theta}}\, d\theta = 0$$

2. Let $f = \frac{1}{z^4+z^2+1}$
   (a) Find the radius of convergence of the Taylor series of f at $z = 1$.
   (b) Explicitly find the constant and linear term of the series.

3. Evaluate the integral
$$\frac{1}{2\pi}\int_\gamma \frac{ze^z}{(z-a)^3}dz$$
Assuming the point a lies inside the simple closed curve γ.

4. Let $f(z)$ be an entire function. Assume $|f(z)| \geq 1$ for all z. Show f is constant.

5. Suppose that $f(z)$ is analytic on $\mathbb{C}$ and that there exists constants $A > 0$ and $B > 0$ such that $|f(z)| \leq A|z|^{1/2} + B$ for all $z \in \mathbb{C}$. What can you say about $f(z)$?

6. Suppose that f is entire and that
$$\lim_{z\to\infty} \frac{f(z)}{z} = 0.$$
Prove that f is constant.

7. Prove that
$$\int_0^{\pi} e^{\cos\theta}\cos(\sin\theta)d\theta = \pi.$$
Hint: consider $\int_\gamma (e^z/z)dz$, where γ is the unit circle.

8. Let $f(z)$ be an entire function such that $|f'(z)| < |f(z)|$ for all $z \in \mathbb{C}$. Show that there exists a constant K such that $|f(z)| < Ke^{|z|}$ for all $z \in \mathbb{C}$.

9. Prove that an entire function with a positive real part is constant. [Prove this at least two different ways].

10. Let f and g be entire functions in the complex plane. Let $a \in \mathbb{R}$ be an arbitrary constant.
    (a) Show that if $[Re(f)]^2 \leq [Im(f)]^2$, for all $z \in \mathbb{C}$, then f is a constant function.
    (b) Show that if for all $z \in \mathbb{C}$,$Ref(z) \leq kReg(z)$ for some real constant k (independent of z). Then there are constants a, b such that
$$f(z) = ag(z) + b.$$

11. Prove or disprove. If f is entire and is bounded on the real axis, then f is constant.

12. Find the Laurent series expansion of the function: $f(z) = e^z/(z+1)^2$ centered at $z = -1$

13. Find the Laurent series of f valid within the annulus $\{z \in \mathbb{C} : 1 < |z-1| < 3\}$ and

$$f(z) = \frac{1}{z(z+2)}.$$